Nonlocal And Photino Contributions To The Leptons Anomalous Magnetic Moments And Some Physical Measurement Constants

Khavtgai Namsrai:
Center for Quantum Science and Technology,
Institute of Physics and Technology, MAS, Ulaanbaatar, Mongolia
Email: n_namsrai@hotmail.com

Abstract— In this paper we calculate leptons AMM, present some Planck type constants and discuss the running spacetime measurement. It is shown that the difference between $\alpha_\mu^{exp} - \alpha_\mu^{SM}$ is explained by means of the nonlocal theory and existence of the photino. We establish exact values of the characteristic lengths $l_{non} = 1.93 \times 10^{-15}\, m$ and $l_{photino} = 1.28 \times 10^{-25} m$, respectively.

Keywords Nonlocal QED, AMM of leptons, photino, the Planck type constants for the universe, characteristic length, zeta function, running spacetime measurement, universe mass.

I. The nonlocal contribution to the leptons anomalous magnetic moment (AMM)

A. The nonlocal contribution to the leptons anomalous magnetic moment (AMM)

a. Introduction

It is well known that cornerstone of ultraviolet divergences in the local quantum electrodynamics (QED) is associated with singularity of the classical electrodynamics. Indeed by using the Yukawa corresponding principle Fourier transform of the Coulomb potential $U_C(r)$:

$$D(\boldsymbol{p}) = \frac{1}{\boldsymbol{p}^2} = \frac{1}{e}\int d^3 r e^{i\boldsymbol{p}r}\left(\frac{e}{4\pi}\frac{1}{r}\right) \qquad (1)$$

gives the photon propagator in the static limit in the momentum space. In the relativistic case it takes the form in x-space:

$$D_{\mu\nu}(x) = \frac{i}{(2\pi)^4} g_{\mu\nu}\int d^4 p e^{ipx}\frac{1}{p^2 + i\varepsilon},$$

$$p^2 = p_0^2 - \boldsymbol{p}^2 \qquad (2)$$

Here $U_C(0) = \infty$ and $D_{\mu\nu}(0) = \infty$. as it should be.

We know that with using this photon propagator excellent theory of QED was developed and its theoretical and experimental results for various physical quantities coincide with enormous accuracy. So that this is the best theory physicists have ever done.

In previous paper [1] we have introduced more simple form of changing of the Coulomb potential

$$U_C(r) \Longrightarrow U_C^l(r) = \frac{e}{4\pi}\frac{1}{\sqrt{r^2 + l^2}}. \qquad (3)$$

and obtained its corresponding nonlocal photon propagator

$$D_{\mu\nu}^l(x) = \frac{i}{(2\pi)^4} g_{\mu\nu}\int d^4 p e^{ipx}\frac{V_l(-p^2 l^2)}{p^2 + i\varepsilon}, \qquad (4)$$

where formfactor $V_l(-p^2 l^2)$ is given by the Mellin representation

$$V_l(-p^2 l^2) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\eta \frac{v(\eta)}{\sin^2 \pi\eta}[l^2(-p^2]^{1+\eta}, \qquad (5)$$

$(1 < \beta < 2)$,

$$v(\eta) = \frac{\pi}{4^{1+\eta}}\frac{1}{\Gamma(1+\eta)\Gamma(2+\eta)}. \qquad (6)$$

In the nonlocal theory:

$$U_C^l(0) = \frac{e}{4\pi}\frac{1}{l}$$

and

$$D_{\mu\nu}^l(0) = g_{\mu\nu}\frac{1}{4\pi^2 l^2}.$$

b. Nonlocal quantum electrodynamics

Lagrangian functions of the nonlocal quantum electrodynamics arisen from the modification of the Coulomb potential at small distances have similar structures as the local theory.

$$L(x) = e{:}\bar\psi(x)\hat A(l,x)\psi(x){:} + e(Z_1 - 1){:}\bar\psi(x)\hat A(l,x)\psi(x){:} - \delta m{:}\bar\psi(x)\psi(x){:} + (Z_2 - 1){:}\bar\psi(x)\left(i\hat\partial - m\right)\psi(x){:} - (Z_3 - 1)\frac{1}{4}{:}F_{\mu\nu}(x)F^{\mu\nu}(x){:}, \qquad (7)$$

where

$$\hat A(l,x) = A_\mu(l,x)\gamma^\mu, \qquad \hat\partial = \gamma^\mu\frac{\partial}{\partial x_\mu}.$$

Only in our case of the nonlocal theory, renormalization constants $Z_1, Z_2, Z_3, \delta m$ are finite and moreover $Z_1 = Z_2$ due to the Ward - Takahashi identity. Here "chronological" pairing (or T - product)

Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 2458-9403
Vol. 11 Issue 6, June - 2024

of the fermionic field operators of electrons has the usual local form:

$$S(x-y) = \langle 0 | T[\psi(x)\bar{\psi}(y)] | 0 \rangle$$
$$= \frac{1}{(2\pi)^4}\frac{1}{i}\int d^4p \frac{e^{-ip(x-y)}}{m-\hat{p}-i\varepsilon}, \qquad (8)$$

while "causal" function of the nonlocal electromagnetic field $A_\mu(l,x)$ in (7) takes the form due to the formula

$$D^l_{\mu\nu}(x-y) = g_{\mu\nu}D^l(x-y)$$
$$= -\frac{g_{\mu\nu}}{(2\pi)^4 i}\int d^4p e^{-ip(x-y)}\frac{V_l(-p^2l^2)}{-p^2-i\varepsilon}, \qquad (9)$$

where $V_l(-p^2l^2)$ is given by formulas (4) and (5).

c. *The Electron Self - Energy in NQED*

The complete electron propagator in NQED is given by the sum

$$[-i(2\pi)^{-4}S'_l(p)]$$
$$= [-i(2\pi)^{-4}S(p)]$$
$$+ [i(2\pi)^{-4}S(p)][i(2\pi)^4\Sigma_l(p)] \times$$
$$\times [-i(2\pi)^{-4}S(p) + \cdots,$$

where

$$S(p) = \frac{m+\hat{p}}{m^2-p^2-i\varepsilon}.$$

The sum is trivial and gives

$$S'_l(p) = [m-\hat{p}-\Sigma_l-i\varepsilon]^{-1}.$$

In lowest order there is a one-loop contribution to Σ_l, given in Fig.1:

$$-i{:}\bar{\psi}(x)\Sigma_l(x-y)\psi(y){:},$$

where

$$\Sigma_l(x-y) = -ie^2\gamma_\mu S(x-y)\gamma_\mu D^l(x-y). \qquad (10)$$

Passing to the momentum representation and going to the Euclidean metric by using $k_0 \to exp(i\pi/2)k_4$, one gets

$$\tilde{\Sigma}_l = \frac{e^2}{(2\pi)^2}\int d^4k_E \frac{V_l(k_E^2l^2)}{k_E^2}\gamma_\mu^{(E)}\frac{m-\hat{p}_E+\hat{k}_E}{m^2+(p_E-k_E)^2}\gamma\mu^{(E)}.$$

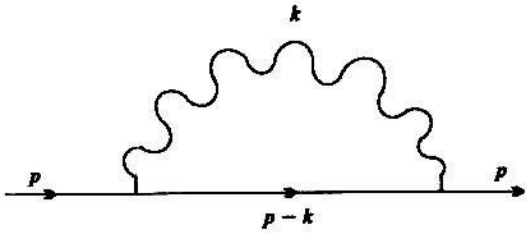

Fig. 1. *Diagram of Self - energy of a electron in NQED*

Here $p_E = (-ip_0, \boldsymbol{p}), \gamma^{(E)} = (-i\gamma_0, \boldsymbol{\gamma})$ and $k_E = (k_4, \boldsymbol{k})$. Taking into account the Mellin representation (5) for the form factor $V_l(k_E^2l^2)$ and after some calculations, we have

$$\tilde{\Sigma}_l(p)$$
$$= -\frac{e^2}{8\pi}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\eta \frac{1}{\sin^2\pi\eta}\frac{v(\eta)(m^2e^2)^{1+\eta}}{\Gamma(2+\eta)}F(\eta,p) \quad (11)$$

where

$$F(\eta,p) = \frac{1}{\Gamma(-\eta)}\int_0^1 du\left(\frac{1-u}{u}\right)^{1+\eta}\left(1-\frac{p^2}{m^2}\right)^{1+\eta}(2m - \hat{p}m) \qquad (12)$$

is a regular function in the half-plane $Re\,\eta > -2$. Assuming the value m^2l^2 to be small, one can obtain (after calculation of residues at the points $\eta = -1{,}0$)

$$\tilde{\Sigma}_l(p) = \frac{e^2}{8\pi^2}\int_0^1 du(2m-\hat{p}u)\ln\left(1-\frac{p^2}{m^2}\right) +$$
$$+\frac{e^2}{8\pi^2}\left\{\left(\ln\frac{m^2l^2}{4}\right)\left(2m-\frac{1}{2}\hat{p}\right)+\hat{p}\left(\frac{1}{2}+\psi(1)\right) - 4m\psi(1)\right\} +$$
$$\frac{me^2}{32\pi^2}(m^2l^2)[ln^2\left(\frac{m^2l^2}{4}\right)-\left(\ln\frac{m^2l^2}{4}\right)(3+4\psi(1)) +$$
$$+4\psi(1)(1+\psi(1))+2-\frac{1}{3}\pi^2], \qquad (13)$$

where $\psi(1) = -C;\ C = 0.57721566490\ldots$ is the Euler number.

d. *Vertex Function and Anomalous Magnetic Moment of Leptons in NQED*

Let us consider Feynman diagram shown in Figure 2. The following matrix element corresponds to this diagram:

$$ie{:}\bar{\psi}(x)\Gamma^l_\mu(x,z/y)\psi(z)A_\mu(y){:}. \qquad (14)$$

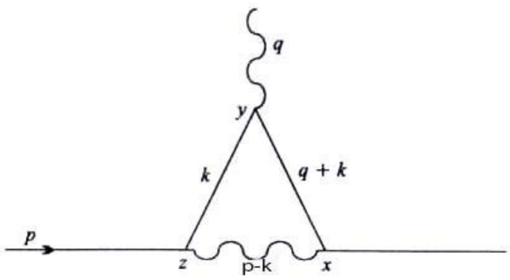

Fig. 2. *Vertex function in NQED*

Analogously, in the momentum space and in the Euclidean metric, the vertex function takes the form

$$\tilde{\Gamma}^l_\mu(p_1,p) = -\frac{e^2}{(2\pi)^4}\int d^4k_E\frac{V_l((p_E-k_E)^2l^2)}{(p_E-k_E)^2}\gamma_\nu \times$$
$$\times\frac{m-\hat{k}_E-\hat{q}_E}{m^2+(k_E+p_E)^2}\gamma_\mu\frac{m-\hat{k}_E}{m^2+{k_E}^2}\gamma_\nu. \qquad (15)$$

Again passing to the Minkowski metric and using the generalized Feynman parameterization formula

$$\frac{1}{a^{n_1}b^{n_2}}$$
$$= \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)}\int_0^1 dx x^{n_1-1}(-x)^{n_2-1}\frac{1}{[ax+b(1-x)]^{n_1+n_2}}$$

one gets

$$\tilde{\Gamma}^l_\mu(p_1;p)$$
$$= -\frac{e^2}{8\pi}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\eta\frac{v(n)}{\sin^2\pi\eta}\frac{(m^2l^2)^{1+\eta}}{\Gamma(2+\eta)}F_\mu(\eta;p_1,p), \quad (16)$$

where

$$F_\mu(\eta;p_1,p) = \gamma_\mu F_1(\eta;p_1,p) + F_2(\eta;p_1,p).$$

Here

$$F_1(\eta;p_1,p)=\frac{1}{\Gamma(-\eta)}\int_0^1\int_0^1\int_0^1 d\alpha d\beta d\gamma\delta(1-\alpha-\beta-\gamma)\alpha^{-1-\eta}Q^{1+\eta},$$

$$F_2(\eta;p_1,p)=\frac{1}{\Gamma(-1-\eta)}\int_0^1\int_0^1\int_0^1 d\alpha d\beta d\gamma\delta(1-\alpha-\beta-\gamma)\alpha^{-1-\eta}Q^{\eta}\times$$
$$\times\frac{1}{m^2}[m^2\gamma_\mu-2mq_\mu+4m(\beta q_\mu-\alpha p_\mu)+(\alpha\hat{p}-\beta\hat{q})\gamma_\mu\hat{q}+$$
$$+(\alpha\hat{p}-\beta\hat{q})\gamma_\mu(\alpha\hat{p}-\beta\hat{q})], \qquad (17)$$
$$Q=\beta+\gamma-\alpha\gamma\frac{p^2}{m^2}-\beta\gamma\frac{q^2}{m^2}-\alpha\beta\frac{(p+q)^2}{m^2}. \qquad (18)$$

Let us calculate the vertex function (16) for two cases: first, when $q=0$ and p has an arbitrary value; second, when q is an arbitrary quantity and p; p_1 are situated on the m-mass shell. In the first case, assuming $q=0$ in the formula (17) and after some standard calculations, one gets

$$F_\mu(\eta;p_1,p)=\frac{1}{\Gamma(-\eta)}\int_0^1 du\left(\frac{1-u}{u}\right)^{1+\eta}\left(1-u\frac{p^2}{m^2}\right)^{1+\eta}\times$$
$$\times\left[u\gamma_\mu+\frac{2(1+\eta)up_\mu(2m-u\hat{p})}{m^2-up^2}\right]. \qquad (19)$$

Comparing this formula with the expression (12) for the self - energy of the electron, it is easily seen that

$$F_\mu(\eta;p_1,p)=-\frac{\partial}{\partial p_\mu}F(\eta;p) \qquad (20)$$

From this identity, we can obtain a very important conclusion. In nonlocal QED constructed by using the modification of the Coulomb potential, the Ward - Takahashi identity is valid:

$$\tilde{\Gamma}_\mu^l(p,p)=-\frac{\partial}{\partial p_\mu}\tilde{\Sigma}_l(p). \qquad (21)$$

In the second case, one can put

$$\bar{u}(\boldsymbol{p}_1)\tilde{F}_\mu^l(p_1,p)u(\boldsymbol{p})=\bar{u}(\boldsymbol{p}_1)\Lambda_\mu(q)u(\boldsymbol{p}), \qquad (22)$$

where $u(\boldsymbol{p}_1)$ and $u(\boldsymbol{p})$ are solutions of the Dirac equation

$$(\hat{p}-m)u(\boldsymbol{p})=0, \qquad \bar{u}(\boldsymbol{p}_1)(\hat{p}_1+m)=0.$$

Substituting the vertex function (16) into (22) and after some transformations, we have

$$\bar{u}(\boldsymbol{p}_1)F_\mu(\eta;p_1,p)u(\boldsymbol{p})$$
$$=u(\boldsymbol{p}_1)\Lambda_\mu(\eta;q)u(\boldsymbol{p}). \qquad (23)$$

Here

$$\Lambda_\mu(\eta;q)=\gamma_\mu f_1(\eta;q^2)+\frac{i}{2m}\sigma_{\mu\nu}q_\nu f_2(\eta;q^2),$$
$$\sigma_{\mu\nu}=\frac{1}{2i}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu),$$
$$f_i(\eta;q^2)=\frac{1}{\Gamma(-\eta)}\int_0^1\int_0^1\int_0^1 d\alpha d\beta d\gamma\delta(1-\alpha-\beta-\gamma)\ \alpha^{1-\eta}L^\eta\times g_j(\alpha,\beta,\gamma,q^2), \quad j=1,2,$$

$$L=\varepsilon\alpha+(1-\alpha)^2-\beta\gamma\frac{q^2}{m^2}, \qquad (24)$$
$$g_1(\alpha,\beta,\gamma,q^2)=(1-\alpha)^2(-\eta)+2\alpha(1+\eta)-\frac{q^2}{m^2}[\beta\gamma+(1+\eta)(\alpha+\beta)(\alpha+\gamma)],$$
$$g_2(\alpha,\beta,\gamma,q^2)=2\alpha(1-\alpha)(1+\eta).$$

To avoid infrared divergences in the vertex function we have introduced here the parameter $\varepsilon=\frac{m_{ph}^2}{m^2}$, taking into account the "mass" of the photon. Finally, one gets

$$\Lambda_\mu(q)=\gamma_\mu F_1(q^2)+\frac{i}{2m}\sigma_{\mu\nu}q_\nu F_2(q^2), \qquad (25)$$

where

$$F_i(q^2)=\frac{e^2}{8\pi}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\eta\frac{v(\eta)}{sin^2\pi\eta}\frac{(m^2l^2)^{1+\eta}}{\Gamma(2+\eta)}f_i(\eta;q^2). \qquad (26)$$

It is easy to verify that the vertex function $\Lambda_\mu(q)$ satisfies the gauge invariant condition:

$$q_\mu\bar{u}(p_1)\Lambda_\mu(q)u(p)=0. \qquad (27)$$

Let us write the first terms of the decomposition for the functions $F_1(q^2)$ and $F_2(q^2)$ over small parameters m^2l^2 and q^2/m^2:

$$F_2(q^2)=-\frac{\alpha}{2\pi}\left[1+\frac{m^2l^2}{6}\left(\ln\frac{m^2l^2}{4}-2\psi(1)+\frac{1}{6}\right)\right]+O\left(\frac{q^2}{m^2}\right), \qquad (28)$$

$$F_1(q^2)=-\frac{\alpha}{4\pi}\{3\left[\ln\frac{m^2l^2}{4}-2\psi(1)-\frac{3}{2}\right]+$$
$$+m^2l^2\left[\ln\frac{m^2l^2}{4}-2\psi(1)-\frac{1}{3}\right]\}+O\left(\frac{q^2}{m^2}\right). \qquad (29)$$

From this first formula we can see that corrections to the anomalous magnetic moment (AMM) for leptons are given by

$$\Delta\mu=\frac{\alpha}{2\pi}\left[1+\frac{m^2l^2}{6}\left(\ln\frac{m^2l^2}{4}+\frac{1}{6}-2\psi(1)\right)\right]. \qquad (30)$$

We seen that the first term in (30) is exactly famous Schwinger correction obtained in local QED. From errors of the experimental values of the AMM of the electron and muon [2-5]

$$\Delta\mu_{exp}^{(e)}=\frac{\mu_e}{\mu_B}-1=\frac{1}{2}(q-2)$$
$$=(1159652180.73(0.28))\cdot 10^{-12} \qquad (31)$$

and

$$\Delta\mu_{exp}^{(\mu)}=\frac{\mu_\mu}{(e\hbar/2m_\mu)}-1=\frac{1}{2}(g_\mu-2)$$
$$=(116592089(63))\cdot 10^{-11}, \quad (32)$$

one gets the following restriction on the value of the universal parameter (or the fundamental length) l:

$$l \leq 4 \cdot 10^{-13} m \; for \; \Delta\mu_{exp}^{(e)}, \quad (33)$$

$$l \leq 2 \cdot 10^{-15} m \; for \; \Delta\mu_{exp}^{(\mu)}. \quad (34)$$

Next by using very high accuracy experiments [6] and standard model (SM) calculations [7] one gets nonlocal contribution to the difference:

$$a_\mu^{exp} - a_\mu^{SM} = (25.1 \pm 5.9)10^{-10}$$
$$= \frac{\alpha}{2\pi} \frac{m_\mu^2 l^2}{6} \left(ln \frac{m_\mu^2 l^2}{4} + \frac{1}{6} - 2\psi(1) \right) \quad (35)$$

This gives transcendental equation for characteristic length l:

$$xlnx = 12.5x10^{-6},$$

where

$$x \cong 1 = 0.9369 m_\mu^2 l^2.$$

Solution of this equation gives

$$l_{non} = 1.0331 \frac{1}{m_\mu} = 1.93x10^{-15} m$$

exactly.

e. Conclusions

Muon does not point like particle and has some innermost structure like ball with radius l_μ. In other words, the local theory associated with the Coulomb potential is valid until at distance $l = 1.93x10^{-15} m \sim \hbar / m_\mu c$. Therefore, disagreement between theoretical calculations and experimental data for the muon AMM maybe solved within the nonlocal theory.

II. PLANCK TYPE CONSTANTS AND BEHAVIOUR OF RUNNING SPACETIME MEASUREMENT

A. Introduction

We know that physical fundamental constants like the velocity of light c, the Newtonian constant G, the Planck constant $\hbar$ and etc. play a vital role in science, technology and society. By using these constants units of measurement for different physical process and phenomena are established very high accuracy. In this paper we would to construct the Planck type fundamental constants arisen from cosmology and high-dimensional spacetime. Also we discuss some an usual way to spacetime measurement process.

B. Planck type constants in cosmology

Long time ago Planck introduced the fundamental length (now named his name)

$$L_{Pl} = \sqrt{\frac{\hbar G}{c^3}} = 1.616228 \times 10^{-35} \, m$$

by using the velocity of light c, the Newtonian constant G and Planck's constant $\hbar$. It seems that now this length plays a vital role in quantum gravitational theory. Aim of this paper is to get other possible fundamental constants.

Notice that the Einstein formula

$$E_U = M_U \cdot c^2 \quad (36)$$

is closed by a new formula:

$$M_U = \frac{c^2}{G} R_U = \frac{c^3}{G} T_U, \quad (37)$$

where c and G are the velocity of light and the Newton constant. Here

$$T_U = (13.787 \pm 0.020) \times 10^9 \; years \quad (38)$$

age of the universe and

$$R_U = c \, T_U = 1.3 \times 10^{26} \, m \quad (39)$$

is the radius of a sphere which includes the whole universe with mass M_U. Then mass of the universe is

$$M_U = 1.755 \times 10^{53} \, kg \, , \quad (40)$$

and its potential energy is

$$E_U = M_U \cdot c^2 = 3.8 \times 10^{66} \, kilo \; calorie \, . \quad (41)$$

It is assumed that this new formula for the universe mass is arisen from the Einstein equation with the Schwarzschild radius $r_{sch} = 2R_U$. It means that whole our universe maybe black hole with the radius $r_{hole} = 2R_U$. Finally, numerical observable mass of the universe is

$$(M_U)_{obser} \sim 1.5 \times 10^{53} \, kg \quad (42)$$

coinciding with the above value. Other modest and naïve assumption is that the equation of motion of the whole Galaxies accelerating moving apart from each other in the Universe reduced at arbitrary points of space in any directions in the sky is given by the following formula

$$F_U = M_U \, a_U \quad (43)$$

where force F_U and acceleration a_U are

$$F_U = \frac{c^4}{G} = 1.21 \times 10^{44} N,$$
$$a_U = \frac{c}{T_U} = 6.89 \times 10^{-10} \, \frac{m}{\sec^2} \quad (44)$$

This drift force F_U maybe arisen from the Big Bang process and played a vital role in the phenomenon of inertia. Moreover, we call this driven force F_U vacuum force which is responsible for expanding universe.

C. Modification of the Newtonian and Coulomb laws in high-dimensional spacetime

It turns out that the Newtonian potential and its corresponding force between two bodies with masses M_1 and M_2 depend on number of spacetime dimensions *D*:

$$U_D^N(r_D) = -G \cdot A^{D-4} \frac{M_1}{r_D^{D-3}},$$
$$\vec{F}_D^N(r_D) = G A^{D-4} \, \frac{M_1 \cdot M_2}{r_D^{D-2}} \vec{n}$$

Here point of view of dimensional argument and conserving invariant form of these two formulas there exists one unique universal parameter named the Planck length. Thus, summation of all contributions to the Newtonian potential due to any spacetime dimensions is given by the formula:

$$U_{full}^N(r) = -\frac{G}{4\pi} \frac{1}{r} \sum_{n=0}^{\infty} \left(\frac{L_{Pl}}{r} \right)^n = -\frac{G}{4\pi} \frac{1}{r} \frac{1}{1 - L_{Pl}/r}$$
$$= -\frac{G}{4\pi} \frac{1}{r - L_{Pl}}$$

From this formula, we see that singularity of the Newtonian potential at the point *r=0* is changed to the point $r = L_{Pl}$ and therefore after Planck area the attractive nature of the Newtonian law becomes repulsive one. From this potential form we conclude remarkable assumption that the black hole singularity arising from the Schwarzschild solution of the Einstein equation is disappeared at the distance *r=0.*

Here the spacetime interval ds^2 takes the form

$$ds^2 = \left(c^2 - \frac{2Gm}{r - L_{Pl}}\right) dt^2 - r^2(\sin^2\theta d\varphi^2 + d\theta^2) - \frac{dr^2}{1 - \frac{2Gm}{c^2(r - L_{Pl})}}$$

Notice that similar situations are valid for the Coulomb law due to any dimensional spacetime and its potential form takes the form

$$U_{full}^{C}(r) = \frac{e}{4\pi}\frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{L_{Pl}}{r}\right)^n = \frac{e}{4\pi}\frac{1}{r - L_{Pl}}$$

We also propose an empirical formula

$$F_{PN} = \frac{G\hbar}{c}\frac{M}{r^3} \qquad (45)$$

and call it a Planck-Newtonian force.

For ten (the string theory) and eleven (M theory) dimensions instead of the formula (45) we have

$$F_{st} = \frac{G^5\hbar^2}{c^{10}}\frac{M^4}{r^8}$$

and

$$F_{Mt} = \frac{G^6\hbar^2}{c^{12}}\frac{M^5}{r^9}$$

respectively. True nature of these forces do not depend on number of spacetime dimensions and therefore

$$F_{st} = F_{Mt}$$

from which one gets remarkable relation (37) where $r = R_U$.

D. *Behaviour of the running spacetime measurement process*

Second remark is concering to our spacetime measurement process. If we assume that spacetime is initiated from Big Bang process than our measurement of time and space is given by the following equivalent series:

$$\left.\begin{matrix} T_1 = 1 + 1 + 1 + \cdots \\ or \\ T_2 = 1 + 2 + 3 + \cdots \end{matrix}\right\} years = (13.787 \pm 0.020) \times 10^9 years) + \cdots \qquad (46)$$

and

$$\left.\begin{matrix} S_1 = 1 + 1 + 1 + \cdots \\ or \\ S_2 = 1 + 2 + 3 + \cdots \end{matrix}\right\} meters = 1.3 \times 10^{26}\, m + \cdots \qquad (47)$$

These series do not depend on the choice of measurement units like second, minute, clock, year, cantimeter, meter and kilometer and so on and only one can required that units will be homogeneous one in series (46) and (47).

Here we choose years and meters as a units of time and space. The following question is arisen: these two equivalent series are finite or infinite in the future. For answer, we use the Riemann zeta function

$$\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s} \qquad Re\ s > 0. \qquad (48)$$

Let us consider analytic continuation of the zeta function.

Consider the definition of the gamma function $\Gamma(s) = \int_0^\infty dt\, e^{-t}\, t^{s-1}$. Let $t \to nt$ in this integral, and use the resulting equation to prove that [8]:

$$\Gamma(s)\zeta(s) = \int_0^\infty dt \frac{t^{s-1}}{e^t - 1}, \qquad Re\ (s) > 1. \qquad (49)$$

Further, using the expansion

$$\frac{1}{e^t - 1} = \frac{1}{t} - \frac{1}{2} + \frac{t}{12} + O(t^2). \qquad (50)$$

One gets

$$\Gamma(s)\zeta(s) = \int_0^1 dt\, t^{s-1}\left(\frac{1}{e^t - 1} - \frac{1}{t} + \frac{1}{2} - \frac{t}{12}\right) + \frac{1}{s-1} - \frac{1}{2s} + \frac{1}{12(s+1)} + \int_1^\infty dt \frac{t^{s-1}}{e^t - 1}\,. \qquad (51)$$

For $Re\ (s) > 0$ we have

$$\Gamma(z) = \int_0^1 dt\, t^{z-1}\left(e^{-t} - \sum_{n=0}^{N}\frac{(-t)^n}{n!}\right) + \sum_{n=0}^{N}\frac{(-1)^n}{n!}\frac{1}{z+n} + \int_1^\infty dt\, e^{-t} t^{z-1}. \qquad (52)$$

Thus, this pole structure of $\Gamma(s)$ gives

$$S_1 = T_1 = \zeta(0) = -\frac{1}{2} \qquad (53)$$

and

$$S_2 = T_2 = \zeta(-1) = -\frac{1}{12} \qquad (54)$$

These two remarkable properties prove that if spacetime has the beginning then it exactly is finite in the future:

$$S_{finite} = T_{finite} = \sum_{n=1}^{N}\left\{\begin{matrix}1 + 1 + 1 + \cdots \\ 1 + 2 + 3 + \cdots\end{matrix}\right\}\begin{pmatrix} meters \\ years \end{pmatrix} \qquad (55)$$

E. *Conclusions*

It turs out that these series are finite in any units of spacetime measurement, where N is a very enormous large finite number. This mathematical proof means that time may be shut down in very far future and therefore our expanding universe steps down slowly.

III. PHOTINO CONTRIBUTION TO THE MUON ANOMALOUS MAGNETIC MOMENT (AMM)

A. *Introduction*

Recently disagreement between analytic calculations and experimental data for the anamolous magnetic moment of the muon gives rise many problems and even more one says beyondscope of the standard model. It turns out that this problem is solved elementarily using concept of a nonlocal or extended object with a size or characteristic length l. In previous paper [9], we have used the Yukawa corresponding principle for an extended charge of rigid string like stick (dipole see Figure 3) with the following potential form

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r}\left(1+\frac{2l}{r}\right),\quad \varepsilon_0 = 1 \qquad (56)$$

and obtained propagator

$$D^l_{\mu\nu}(x) = \frac{-i}{(2\pi)^4} g_{\mu\nu}\int d^4p e^{ipx}\left[\frac{1}{p^2-i\varepsilon} + \pi l\,\frac{\sqrt{p^2-i\varepsilon}}{p^2-i\varepsilon}\right], \qquad (57)$$

which consists photon and photino propagators. It was easily to construct nonlocal theory and calculate its primitive Feynman diagrams.

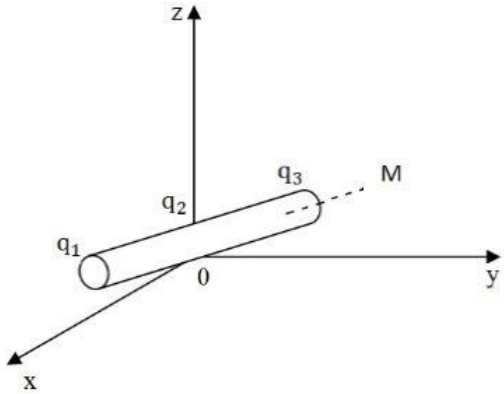

Fig. 3. One dimensional extended charge configuration consisting of three charges with different signs (two plus charges and one minus charge and vice versa.)

The interaction Lagrangian and its S-matrix are constructed by using usual formulas [for detail, see [9]]:

$$L_{in} = ie\bar{\psi}(x)\,\gamma^\mu A_\mu(x)\,\psi(x)$$

and

$$S = T\exp\{i\int d^4x\, L_{in}\, x\}$$

Here

$$\langle 0|\mathrm{T}\{\mathrm{A}_\mu(\mathrm{x})\mathrm{A}_\nu(\mathrm{y})\}|0\rangle = \mathrm{D}_{\mu\nu}(\mathrm{x}-\mathrm{y}) \qquad (58)$$

is the Green or causal function of photon and photino together which is given by formula (57) and

$$\langle 0|\mathrm{T}\{\bar{\psi}(x)\psi(y)\}|0\rangle = \mathrm{S}(\mathrm{x}-\mathrm{y}) \qquad (59)$$

is the usual spinor propagator.

Further, by using the perturbation theory one can calculate S- matrix elements up to any order of the fine structure constant $\alpha = e^2/4\pi$. Here we calculate some primative Feynman diagrams with using propagators (58) and (59).

B. Vacuum polarization

Since in our scheme the spinor propagator (59) does not changed and therefore the diagrams of the vacuum polarization i.e., closed spinor propagators of leptons in the nonlocal QED are studied by the same way as in the local theory

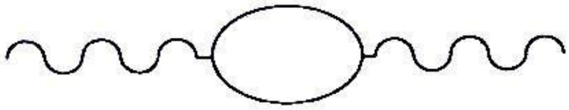

Fig. 4. The one loop diagram for the vacuum polarization in the nonlocal QED arisen from an extended charge

C. Electron self-energy in the nonlocal QED

The matrix element of an one-loop contribution to Σ_l is given by formula (Figure 5)

$$:\bar{\psi}(x)\Sigma_l(x-y)\psi(y):$$

where

$$\Sigma_l(x-y) = --ie^2 D_l(x-y)\boldsymbol{\gamma}^{\boldsymbol{\mu}} S(x-y)\gamma_\mu$$

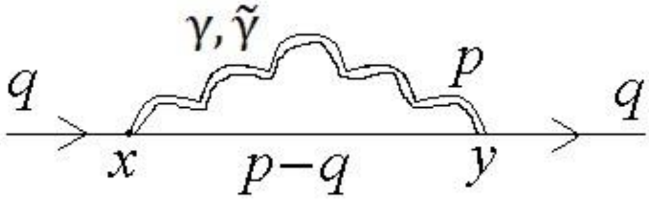

Fig. 5. The one loop diagram for the electron self-energy function in NQED.

In the momentum space takes the form

$$\Sigma_l(q) = \frac{ie^2}{(2\pi)^4}\int d^4p\left[\frac{1}{p^2-i\varepsilon}+\frac{\pi l}{\sqrt{p^2-i\varepsilon}}\right] \times\left[\frac{\gamma^\rho(-i\hat{q}+i\hat{q}+m)\gamma_\rho}{(q-p)^2+m^2-i\varepsilon}\right], \qquad (60)$$

Next we go to the Wick rotation and using the d-dimensional regularization procedure we obtain

$$\Sigma_l(q) = \frac{e\pi^{\frac{d}{2}}\Gamma^2(d/2)}{(2\pi)^4}\int_0^1 dx\ [-i(2-d)(1-x)\hat{q}+md] \times\{\Gamma\left(2-\frac{d}{2}\right)[q^2x(1-x)+m^2x]^{\frac{d}{2}-2} \qquad (61)$$

$$+\frac{\pi l}{2}\frac{\Gamma\left(\frac{3}{2}-\frac{d}{2}\right)}{\Gamma\left(\frac{3}{2}\right)\sqrt{1-x}}[q^2x(1-x)+m^2x]^{\frac{d}{2}-\frac{3}{2}}\} = \Sigma_{local}(q)+\Sigma'_l(q)$$

Here

$$\Sigma_{local}(q) = -\frac{2e^2\pi^2}{(2\pi)^4}\int_0^1 dx\left\{2\frac{1-x^2}{x}(i\hat{q}+m) + (i\hat{q}(1-x) + 2m)\ln\frac{m^2x^2}{q^2x(1-x)+m^2x}\right\} \qquad (62)$$

And

$$\Sigma'_l(q) = -\frac{2e^2\pi^2}{(2\pi)^4}\int_0^1 dx\,(i\hat{q}(1-x)+2m)(-2\pi l) \times\frac{1}{\sqrt{x(1-x)}}\frac{1}{\sqrt{q^2(1-x)+m^2}} \qquad (63)$$

Now we use the integral

$$\int_0^1 x^{\lambda-1}(1-x)^{\mu-1}(1-\beta x)^{-\nu}dx = B(\lambda,\mu)\,{}_2F_1(\nu,\lambda;\lambda+\mu;\beta)$$

and obtain the hypergeometric form of $\Sigma'_l(q)$:

$$\Sigma'_l(q) = \frac{e^2 l}{2}\frac{1}{\sqrt{m^2+q^2}}\left[\frac{i}{4}\hat{q}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2};2;\beta\right) + m\,{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;\beta\right)\right] \quad (64)$$

where

$$\beta = \frac{q^2}{m^2+q^2}, \qquad q^2 \neq -m^2$$

D. Anomalous magnetic moments of the leptons

In the nonlocal QED, one-loop graph (Figure 6) gives the following matrix element

$$\Gamma_l^{\mu}(p',p) = \int d^4k[e\gamma^{\rho}(2\pi)^4]\left[\frac{-i}{(2\pi)^4}\frac{-i(\hat{p}'-\hat{k})+m}{(p'-k)^2+m^2-i\varepsilon}\right] \times [\gamma^{\mu}[\left[\frac{-i}{(2\pi)^4}\frac{-i(\hat{p}-\hat{k})+m}{(p-k)^2+m^2-i\varepsilon}\right][e\gamma_{\rho}(2\pi)^4] \times \left[\frac{-i}{(2\pi)^4}\left(\frac{1}{k^2-i\varepsilon}+\pi l\frac{1}{\sqrt{k^2-i\varepsilon}}\right)\right] \quad (65)$$

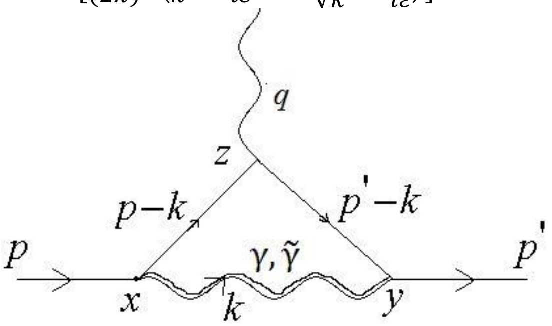

Fig. 6. One-loop diagram for the (nonlocal) photons-lepton vertex function Γ^{μ} in NQED.

General form of this vertex function is

$$\bar{u}(p')\Gamma_l^{\mu}(p',p)u(p) = \bar{u}(p')\left[\gamma^{\mu}F_l(q^2) - \frac{i}{2m}(p+p')^{\mu}G_l(q^2)\right]u(p) \quad (66)$$

Here we have interested in second term

$$G_l(q^2) = G_{local}(q^2) + G_{1l}(q^2) \quad (67)$$

where

$$G_{local}(q^2) = -\frac{e^2\,m^2}{4\pi^2}\int_0^1 dx\int_0^x dy\frac{x(1-x)}{m^2x^2+q^2y(x-y)} \quad (68)$$

and

$$G_{1l}(q^2) = -\frac{e^2\,m^2}{4\pi^2}l\int_0^1 dx\frac{1}{\sqrt{1-x}} \times \int_0^x dy\frac{x(1-x)}{\sqrt{m^2x^2+q^2y(x-y)}} \quad (69)$$

Last two terms are finite and leading to AMM of leptons:

$$\mu_l = \frac{e}{2m}[1-G_{local}(0)-G_{1l}(0)] \quad (70)$$

where

$$-G_{local}(0) = \frac{\alpha}{2\pi} \quad (71)$$

$$-G_{1l}(0) = \frac{4}{15}\alpha l m \quad (72)$$

The contribution (71) is the famous α/2π correction first obtained by Schwinger and the second term (72) is due to photino contribution to AMM of leptons. The present experimental values of the anomalous magnetic moment of the electron and muon (Carey et.al., 1999 [10], Particle Data Group,2002 [11]; Barger et.al.,2005 [12]; Yao, 2006 [13] are given by expressions

$$\Delta\mu_{exp}^{(e)} = \frac{\mu_e}{\mu_B} - 1 = \frac{1}{2}(q-2) = (1159652180.73(0.28))\cdot 10^{-12}$$

and

$$\Delta\mu_{exp}^{(\mu)} = \frac{\mu_{\mu}}{(e\hbar/2m_{\mu})} - 1 = \frac{1}{2}(g_{\mu}-2) = (116592089(63))\cdot 10^{-11},$$

(see, for example, Heinemeyer et al., 2004 [14]). Using errors of these experimental data and the calculated photino contribution (72) one can establish the following restrictions on a scale of the characteristic length or size of the rigid string stick:

$$l \leq \begin{cases} 2.96\cdot 10^{-27}m\ for\ e, \\ 3.22\cdot 10^{-26}m\ for\ \mu \end{cases} \quad (73)$$

Recent very high accuracy experimental result [6] for measuring the anomalous magnetic moment of muon

$$a_{\mu}(FNAL) = 11659055\ (24)x10^{-11}\ (0.20\ ppm)$$

and

$$a_{\mu}(exp) = 11659059\ (22)x10^{-11}\ (0.19\ ppm)$$

allows us to compare with Standart Model (SM) calculation [7]

$$a_{\mu}^{SM}(theory) = 116591810\ (43)x10^{-11}.$$

As a result difference of these two values is

$$\Delta = a_{\mu}^{exp} - a_{\mu}^{SM} = (25.1 \pm 5.9)10^{-10}.$$

From the formula (72) we see that this difference is given by the following formula

$$\Delta = \frac{4}{15}\boldsymbol{\alpha} m_{\mu} l.$$

E. Conclusion

Thus the characteristic length of the nonlocal theory with photino arisen from the extended electric charge is given by value

$$l_{phot} = 1.28x10^{-25}m$$

exactly.

We notice that disagreement between theoretical calculations in SM and experimental data for the muon AMM can be easily solved in the framework of extended charge which radiates photon and photino simulatanuosly.

REFERENCES

[1] Namsrai, Kh. Nonlocal, Quantum Electrodynamics (2019). Journal of scientific and engineering research, 6(8): 195-205, ISSN: 2394-2630.

[2] D. Hanneke, S. Fogwell, and G. Gabrielse, New Measurement of the Electron Magnetic Moment and the Fine Structure Constant 2008 Phys. Rev. Lett. **100**, 120801.

[3] D. Hanneke, S. Fogwell Hoogerheide and G. Gabrielse, Cavity control of a single-electron quantum cyclotron: Measuring the electron magnetic moment, 2011 Phys. Rev. A **83**, 052122, DOI:https://doi.org/10.1103/PhysRevA.83.052122.

[4] Bennett, G.W. et.al. Measurement of the Negative Muon Anomalous magnetic moment to 0.7 ppm, 2004, Phys. Rev. Lett. **92**, 161802, DOI:https://doi.org/10.1103/PhysRevLett.92.161802.

[5] Roberts, B.L. (2010), Chinese Phys. C34, 741. Aoyama,T. et.al. (2012), Tatsumi Aoyama, Masashi Hayakawa, Toichiro Kinoshita and Makiko Nio, Tenth-Order QED contribution to the electron g-2 and an improved value of the Fine Structure constant, Phys. Rev. Lett., **109**, DOI:https://doi.org/10.1103/PhysRevLett.109.111807. 111808; Phys. Rev. D85, 033007.

[6] Aguillard, D.P, et al, Measurement of the Positive Muon Anomalous Magnetic Moment to 0.20 ppm (2023), DOI:https://doi.org/10.1103/PhysRevLett.131.161802. [Muon g-2]. Phys. Rev. Lett., **131**, №16, 161802, Abi, B. et al (2021) [Muon g-2]. Phys. Rev. Lett., 126, №14, 141801

[7] Aoyama, T. et al. (2020). The anomalous magnetic moment of the muon in the SM., Phys. Rept. Vol. 887, Pages 1-166, https://doi.org/10.1016/j.physrep.2020.07.006.

[8] B. Zwiebach (2004). A First Course in String Theory, Cambridge University Press, pp. 63, 243, https://doi.org/10.1017/CBO9780511841620.

[9] Namsrai, Kh. (2020), Extended Charge and Indication of a Scale of the Supersymmetry, Journal of Scientific and Engineering Research, 7(3), 131- 149

[10] Carey, R.M. et al. New Measurement of the Anomalous Magnetic Moment of the Positive Muon (1999). Phys. Rev. Lett. **82,** 1632, https://doi.org/10.1103/PhysRevLett.82.1632

[11] Particle data group, Review of Particle Properties (2002). Phys. Rev. **D66** (1-I), 010001-25, https://doi.org/10.1103/PhysRevD.66.010001.

[12] Barger. V., Kao C., Langacker, P., and Lee, H.S., Muon Anomalous Magnetic Moment in a Supersymmetric U(1)′ Model (2005). Phys. Lett. **B614**, 67-77.

[13] Yao, W. -M. et al. (2006). (Particle Data Group, 2007). Journal of Physics G, 33, 1.

[14] Heinmeyer, S., Stockinger, D., and Weiglein, G., Electroweak and supersymmetric two-loop corrections to (_{g-2})μ (2004). Nuclear Physics, **B699**, 103-123, http://adsabs.harvard.edu/cgi-bin/nph-bib_query?bibcode=2004NuPhB.699..103H&db_key=PHY