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Nonlocal Quantum Field Theory Arisen From High Dimensional Spacetime

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Abstract— We construct nonlocal quantum electrodynamics arisen from high dimensional spacetime and investigate primitive Feynman diagrams. We also calculate restriction on the value of the fundamental length l.

Keywords Quantum electrodynamics, Planck length, Coulomb, Yukawa potentials, leptons anomalous magnetic moment

I. Introduction

We know that from the beginning of early development of the quantum field theory ultraviolet divergent problem encountering in it acquired central place and attempts its solutions have been done by many numerous authors. Recently, in spite of titanic strengthening in elimination of ultraviolet divergences in calculations of matrix elements of S-matrix in field theory, until now full and satisfactory solutions of this problem are absent. By Feynman's joke, for the time being, we put it under a nice carpet.

Among many attempts for solving ultraviolet divergences, like beginning of primitive high momentum cut-off, Paulli-Villars regularization procedure, introducing formfactor cut-off, or cyclic regularization of vacuum polarization diagrams and so on, gauge invariant unification of all forces in nature, in particular, electroweak unification and the string theory connected with a "size" of particles like strings play a vital role in solving devergent problems in interaction mechanisms between elementary particles. Moreover, introducing superfields which are mixed bosons and fermions lead to cancel their closed loops altogether.

In this work, we propose threefold way to elimination of ultraviolet divergences in S-matrix elements by using more primitive ides:

-The first:

Initial roots of this difficult problem belong to the classical physical level, when potential forces like Coulomb, Newtonian and Yukawa potential acquire singularity at the point $r = 0$. Indeed, Fourier transform by using the Yukawa procedure and idealized concept about structureless point-like particle's potentials $U_C(r), U_N(r), U_Y(r)$ are related in the static limit with corresponding force carrying particle's propagators of photon, graviton and scalar particle with mass m:

$$D_{\gamma,g,Y}(p) = \frac{1}{(2\pi)^3}\int d^3r e^{i\vec{p}\vec{r}} U_{C,N,Y}(r) \Rightarrow \begin{Bmatrix} \dfrac{1}{\vec{p}^{\,2}} \to \dfrac{1}{p_0^2 - \vec{p}^{\,2} - i\epsilon} \\ \dfrac{1}{m^2 + \vec{p}^{\,2}} \to \dfrac{1}{m^2 - p_0^2 + \vec{p}^{\,2} - i\epsilon} \end{Bmatrix}$$

It means that classical level, singularities automatically pass to calculations of S-matrix elements. Therefore, this is one reason of presence of ultraviolet divergences in quantum physics.

- The second:

According to above situation and concrete calculation propose, we assume that for some cases elementary particles instead of strings behavior, also have other simple structure like rigid spheres or rigid sticks. For example, if a changed particle is considered as a rigid stick then its propagator is changed according to above procedure [29].

-The third:

Recently, according to ten-dimensional string theory and eleven-dimensional M-theory, high-dimensional spacetime plays a vital role in physical theories. In this connection, there are some interests in studying corrections arisen from high-dimensional spacetime to physical quantities like different potential, equations and propagators of force carrying particles and to construct quantum field theory within framework of existence of fundamental length like Planck one. Roughly speaking, if we allow to introduce Planck length into physical calculations then it means that we can taking into account gravitational effects into physical quantities through the Newtonian constant G intering into the Planck length. This program is realized in section 2-3. It turns out that mathematical calculations of physical quantities in any dimensional case possess remarkable and beautiful universal characters independing on numbers of high-dimensional spacetime. These properties will be shown in these sections.

In conclusion, mathematical bases of this work are presented in Namsrai [24], [25] and [26]. Also historically, problems discussed in this review article were originated in the book "Theoretical physics in the twentieth century" (A memorial volume to W.Pauli) [27].

II. MATHEMATICAL CALCULATION

A. *Fourier Transformation of Physical potentials in d-dimensional space*

To construct physical theory in any D-dimensional Minkowski space ($D = x_0 = ct, d$ -space dimension it should be consider, in first d-dimensional Euclidean space and study Fourier transform of physics quantities in interest. For example, it is well known that in the static limit the Coulomb and the Yukawa potentials are related with a photon propagator and a propagator of a scalar particle with mass m by the following formulas:

$$U_C(r) = \frac{e}{(2\pi)^3}\int \frac{d^3p}{\vec{p}^2} e^{i\vec{p}\cdot\vec{r}} = \frac{e}{4\pi}\frac{1}{r} \qquad (1)$$

and

$$U_Y(r) = \frac{g}{(2\pi)^3}\int \frac{d^3p}{m^2 + \vec{p}^2} e^{i\vec{p}\cdot\vec{r}} = \frac{g}{4\pi}\frac{1}{r}e^{-mr} \qquad (2)$$

In order to obtain the Newtonian potential by using the graviton propagator we use its form for Minkowski space in general relativity [1]:

$$G_{\alpha\beta,\mu\nu} = \frac{P^2_{\alpha\beta,\mu\nu}}{p^2} - \frac{P^0_{s,\alpha\beta,\mu\nu}}{2p^2} = \frac{\Delta_{\alpha\beta,\mu\nu}}{p^2}$$
$$= \frac{1}{p^2}\cdot\left[g_{\alpha\mu}g_{\beta\nu} + g_{\beta\mu}g_{\alpha\nu} - \frac{2}{D-2}\cdot g_{\mu\nu}g_{\alpha\beta}\right] \qquad (3)$$

where D is the number of spacetime dimensions, P^2 is the transverse and traceless spin 2 projection operator and P^0_s is a spin zero scalar multiplet. The graviton propagator for (Anti) de Sitter space is

$$G = \frac{P^2}{2H^2 - \Box} + \frac{P^0_s}{2(\Box + 4H^2)},$$

where H is the Hubble constant. Note that upon taking the limit $H \to 0$ and $\Box \to -p^2$, the AdS propagator reduces to the Minkowski propagator [2].

As shown in formulas (1) and (2), further, we are not interested in numerator in the form (3). Thus, the Newtonian potential for a body with mass M is given by the formula:

$$U_N(r) = \frac{G\cdot M}{2\pi^2}\int \frac{d^3p}{\vec{p}^2} e^{i\vec{p}\cdot\vec{r}} = \frac{GM}{r}, \qquad (4)$$

Where

$$G = 6.6743 \times 10^{-11}\frac{m^3}{kg\cdot sec^2} \qquad (5)$$

is the Newtonian constant.

D -dimensional case the formulas (1) and (4) takes the form

$$U_D(r) = A\int \frac{d^{D-1}p}{\vec{p}^2} e^{ipr} \qquad (6)$$

where A is a some constant of normalization,

$$d^{D-1}p = p^{D-2}dpd\varphi\sin\theta_1 d\theta_1$$
$$\cdot\sin^2\theta_2 d\theta_2 \ldots \sin^{D-3}\theta_{D-3}d\theta_{D-3} =$$
$$= p^{D-2}dpd\varphi\prod_{k=1}^{D-3}\sin^k\theta_k d\theta_k, \qquad (7)$$

here

$$(0 < p < \infty, \quad 0 < \varphi < 2\pi, \quad 0 < \theta_k < \pi),$$
$$\left.\begin{aligned} pr &= p_1x_1 + p_2x_2 + \cdots p_{D-1}\cdot x_{D-1}, \\ p^2 &= p_1^2 + p_2^2 + \cdots + p_{D-1}^2. \end{aligned}\right\} \qquad (8)$$

Here, we use the following general integrals [3]:

$$\int_0^1 d\,x\cos(ax)(1-x^2)^{\nu-\frac{1}{2}}$$
$$= \frac{\sqrt{\pi}}{2}\left(\frac{2}{a}\right)^\nu \Gamma\left(\nu + \frac{1}{2}\right)J_\nu(a) \qquad (9)$$

and

$$\int_0^\infty d\,xx^\mu J_\nu(ax) = 2^\mu a^{-\mu-1}\frac{\Gamma\left(\frac{1}{2}+\frac{1}{2}\nu+\frac{1}{2}\mu\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{2}\nu-\frac{1}{2}\mu\right)} \qquad (10)$$
$$-Re\nu - 1 < Re\mu < \frac{1}{2}, \quad a > 0,$$

where $J_\nu(x)$ is the Bessel function of the order ν.

After some calculations we obtain universal formula for the Newtonian potential and its corresponding force between two bodies with masses M_1 and M_2 for any D -dimensions [4]:

$$U_D^N(r_D) = -G\cdot A^{D-4}\frac{M_1}{r_D^{D-3}}, \qquad (11)$$

$$\vec{F}_D^N(r_D) = \frac{1}{D-3}\vec{\nabla}_D U_D^N(r_D)M_1\cdot M_2$$
$$= GA^{D-4}\frac{M_1\cdot M_2}{r_D^{D-2}}\vec{n}, \qquad (12)$$

Here point of view of dimensional argument and conserving invariant forms of (11), (12) there exists one unique universal parameter named the Planck length, we call it the Planck length

$$A = L_{Pl} = \sqrt{Gh/c^3}. \qquad (13)$$

Thus, summation of all constributions to the Newtonian potential due to any spacetime dimensions is given by the formula:

$$U_{full}^N(r) = -\frac{G}{4\pi}\frac{1}{r}\sum_{n=0}^\infty\left(\frac{L_{Pl}}{r}\right)^n = -\frac{G}{4\pi}\frac{1}{r}\frac{1}{1 - L_{Pl}/r}$$
$$= -\frac{G}{4\pi}\frac{1}{r - L_{Pl}}. \qquad (14)$$

From this formula, we see that singularity of the Newtonian potential at the point $r=0$ is changed to the point $r=L_{Pl}$ and therefore after the Planck area the attractive nature of the Newtonian law becomes repulsive one.

Notice that similar situation are valid for the Coulomb law due to any dimensional spacetime and its potential form takes the form

$$U_{full}^{C}(r)=\frac{e}{4\pi}\frac{G}{4\pi}\frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{L_{Pl}}{r}\right)^{n}=\frac{e}{4\pi}\frac{1}{r-L_{Pl}} \qquad (15)$$

Now we turn to calculate the Yukawa potential in any D-dimensional spacetime and for this case the formula (6) takes the form

$$U_{D}^{Y}(r)=C\int\frac{d^{D-1}p}{m^{2}+p^{2}}e^{ipr}, \qquad (16)$$

where c is a some normalization constant. For calculation purpose, we use the formula (9) and the following integral form [3]:

$$\int_{0}^{\infty}d\,x\frac{J_{\nu}(bx)x^{1+\nu}}{(a^{2}+b^{2})^{1+\mu}}=\frac{a^{\nu-\mu}b^{\mu}}{2^{\mu}\Gamma(1+\mu)}K_{\nu-\mu}(ab), \qquad (17)$$

where

$$-1<Re\nu<Re\left(2\mu+\frac{3}{2}\right),\quad a,b>0$$

and $K_{\nu}(x)$ is the Mack'Donald function.

After some calculations, we have

$$U_{D}^{Y}(r_{D})=g_{D}^{Y}\left(\frac{m}{r_{D}}\right)^{\left(\frac{D-3}{2}\right)}K_{\frac{D-3}{2}}(mr_{D}), \qquad (18)$$

here g_{D}^{Y} is coupling constants for the Yukawa theory.

For our usual four-dimensional spacetime $D=4$, one gets the Yukawa potential form (2), since

$$K_{1/2}(r)=\sqrt{\frac{\pi}{2r}}e^{-r} \qquad (19)$$

Notice that the Newtonian and the Coulomb potentials

$$U_{D}^{N,C}(r)\sim const\cdot\frac{1}{r^{D-3}} \qquad (20)$$

depending on spacetime dimensions satisfy the Laplacian equation

$$\Delta_{D}U_{D}^{N,C}(r_{D})=0, \qquad (21)$$

where

$$\Delta_{D}=\frac{\partial^{2}}{\partial x_{1}^{2}}+\frac{\partial^{2}}{\partial x_{2}^{2}}+\cdots+\frac{\partial^{2}}{\partial x_{D-1}^{2}}$$

and

$$r_{D}=\sqrt{x_{1}^{2}+x_{2}^{2}+\cdots+x_{D-1}^{2}}. \qquad (22)$$

B. High-Dimensional Spacetime and Changing Propagators of Force Carrying Particles

In previous section, we have been shown that potential forces between different fields are changed depending on numbers of spacetime dimensions. It means that force carrying particle's propagator are also must be changed due to high dimensional spacetime from lower dimensional case. First, let us consider five-dimensional space-time which consists from addendum (or addition) of included four-and covering one-dimensional spaces, like $D_4\cup D_1$, in other words, four-dimensional space is embodied into five-dimensions and its co-ordinate points of events are denoted by symbols:

$$x^{\mu}=\left(x_{0}=ct,x_{1},x_{2},x_{3},x_{5}=L_{Pl}a\,\vec{n}\right), \qquad (23)$$

where a is dimensionless parameter or number of lattice in fifth-direction and $\vec{n}$ is an arbitrary unit directing vector. Then the metric tensor is

$$g^{\mu\nu}=\begin{pmatrix}1&0&0&0&0\\0&-1&0&0&0\\0&0&-1&0&0\\0&0&0&-1&0\\0&0&0&0&-1\end{pmatrix} \qquad (24)$$

and an interval reads

$$S^{2}=g^{\mu\nu}x_{\mu}x_{\nu}=x_{0}^{2}-\vec{x}^{2}-L_{Pl}^{2}a^{2}. \qquad (25)$$

Without loss of generality, if we assume $a=1$, in (23) then such type of five-dimensional spacetime was considered by Markov [5]. D'Alembertian in five-dimensional spacetime takes the form

$$\Delta_{5}=-\frac{1}{c^{2}}\frac{\partial^{2}}{\partial t^{2}}+\frac{\partial^{2}}{\partial\,\vec{x}^{2}}+\frac{\partial^{2}}{L_{Pl}^{2}\partial a^{2}}. \qquad (26)$$

We propose following type of wave equation for a scalar particle

$$(\Delta_{5}-m^{2})\Phi(x^{\mu})=0. \qquad (27)$$

Then plane-wave solution of this equation is given by

$$\Phi(x^{\mu})=A\cdot e^{ix_{i}\cdot p^{i}-ip^{5}x_{5}} \qquad (28)$$

where

$$p^{5}=\sqrt{m^{2}-p^{2}}=\sqrt{m^{2}-p_{0}^{2}+\vec{p}^{2}},\qquad\vec{n}\cdot\vec{n}=1,$$

$$ix_{\mu}p^{\mu}=ix_{0}p^{0}-i\,\vec{x}\,\vec{p}-i\sqrt{m^{2}-p^{2}}L_{Pl}a\,\vec{n} \qquad (29)$$

Then direct calculation for equation (27) gives

$$p_{0}^{2}-\vec{p}^{2}-\left(m^{2}-p_{0}^{2}+\vec{p}^{2}\right)-m^{2}=0,$$

or

$$2\left(p_{0}^{2}-\vec{p}^{2}-m^{2}\right)=0 \qquad (30)$$

It means that motion of equation for free particles does not charge in five-dimensional spacetime case, where

$$E = \pm\sqrt{m^2 + \vec{p}^{\,2}}$$

is valid as for Klein-Gordon case.

An inverse Fourier transforms with respect to formulas (1), (2) and (6) for photon, graviton and scalar particle propagators lead to those modifications.

For example, for the photon propagator in static limit in the momentum space one gets

$$\frac{1}{\vec{p}^{\,2}} \Rightarrow D_g^{\gamma}\left(\vec{p}^{\,2}\right) = C\int_0^{\infty}\frac{dr\cdot r^3}{r^2}\int_0^{2\pi}d\,\varphi \times \int_0^{\pi}d\,\theta\sin\theta e^{ipr\cos\theta}\int_0^{\pi}d\,\theta_1\sin^2\theta_1 e^{iL_{Pl}\sqrt{\vec{p}^{\,2}}\cos\theta_1}, \qquad (31)$$

where C is a normalization constant. Then direct calculations gives [6]:

$$D_g^{\gamma}\left(\vec{p}^{\,2}\right) = \frac{1}{\vec{p}^{\,2}} \times \frac{J_1\left(\sqrt{\vec{p}^{\,2}}\,L_{Pl}\right)}{\sqrt{\vec{p}^{\,2}}\,L_{Pl}}. \qquad (32)$$

In formula (31) we have used expression (29) with $a = 1$, where $\vec{n} = \cos\theta_1$ is the directed cosine. Last integral reads

$$I_3 = \int_0^{\pi}d\,\theta_1\cdot\sin^2\theta_1 e^{ipL_{Pl}\cos\theta_1} = \int_{-1}^{1}d\,\lambda\sqrt{1-\lambda^2}e^{ipL_{Pl}\cdot\lambda} = 2\int_0^1 d\,\lambda\sqrt{1-\lambda^2}\cos(pL_{Pl}\lambda) = \frac{\pi}{pL_{pl}}J_1(pL_{Pl}). \qquad (33)$$

Here $J_1(x)$ is the Bessel function of the order 1.

It is obviously that in formula (31) we have used the following integrals:

$$I_1 = \int_0^{\pi}d\,\theta\cdot\sin\theta\cdot e^{ipr\cos\theta} = 2\frac{\sin pr}{pr}, \qquad (34)$$

$$I_2 = \int_0^{\pi}d\,r\sin pr = \frac{1}{p}. \qquad (35)$$

Notice that for a scalar particle it should be change $p = |\vec{p}| \to \sqrt{m^2 + \vec{p}^{\,2}}$ and last integral (35) takes the form

$$I_2^{\gamma} = \frac{1}{p}\int_0^{\infty}dr\, e^{\{-mr\}}\sin pr = \frac{1}{p}\cdot\frac{1}{\sqrt{\{m^2+\vec{p}^2\}}}sin\left(\arctan\frac{p}{m}\right) = \frac{1}{m^2+\vec{p}^2}, \qquad (36)$$

where

$$arctg\, x = arctg\, x\ = arcsin\frac{x}{\sqrt{\{1+x^2\}}}.$$

Finally, photon and graviton propagators in usual four dimensional spacetime, which are arisen from intermediate five-dimensional space take the forms $\left(\vec{p}^{\,2} \to -p^2 = -p_0^2 + \vec{p}^{\,2}\right)$:

$$D_{\mu\nu}^{g}(x) = \frac{g_{\mu\nu}}{(2\pi)^4 i}\int d^4p\frac{V_1(-p^2L_{Pl}^2)}{-p^2-i\varepsilon}e^{ipx}, \qquad (37)$$

$$D_{\mu\nu,\rho\delta}^{g}(x) = \left[g_{\mu\rho}g_{\nu\delta} + g_{\nu\rho}g_{\mu\delta} - \frac{2}{D-2}g_{\mu\nu}g_{\rho\delta}\right]\times \frac{1}{(2\pi)^4 i}\int d^4p\frac{V_1(-p^2L_{Pl}^2)}{-p^2-i\varepsilon}e^{ipx}. \qquad (38)$$

For the Yukawa potential case corresponding scalar particle propagator acquires the form (See: also Section b):

$$D_m^{\gamma}(x) = \frac{1}{(2\pi)^4 i}\int d^4p\frac{V_m(-p^2L_{Pl}^2)}{m^2-p^2-i\varepsilon}, \qquad (39)$$

Here, see formula (43).

For form factors $V(-p^2L_{Pl}^2)$ and $V_m(-p^2L_{Pl}^2)$ the following Mellin representations are valid

$$V(-p^2L_{Pl}^2) = \frac{1}{4i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\,\xi\frac{\left(-\frac{p^2L_{Pl}^2}{4}\right)^{\xi}}{\sin\pi\xi\cdot\Gamma(1+\xi)\cdot\Gamma(2+\xi)}, \qquad (40)$$

$$V_m(-p^2L_{Pl}^2) = \frac{1}{4i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\,\xi\frac{\left[\left(\frac{m^2-p^2}{4}\right)L_{Pl}^2\right]^{\xi}}{\sin\pi\xi\cdot\Gamma(1+\xi)\cdot\Gamma(2+\xi)}, \qquad (41)$$

Now let us calculate the scalar particle propagator leading to formulas (39) and (41).

Thus, for the Yukawa potential case, when four dimensional space is embodied into five dimensions

$$U_4^{\gamma}(r) = \frac{g_4}{r}e^{-mr} = g'_5\left(\frac{m}{r}\right)^{1/2}K_{1/2}(mr)$$

and therefore

$$D_S^{\gamma} = const\int_0^{\infty}d\,r\cdot r^{\frac{3}{2}}K_{\frac{1}{2}}(mr)\frac{\sin pr}{r}\times \int_0^{\pi}d\,\theta_1\sin^2\theta_1 e^{i\sqrt{\vec{p}^{\,2}+m^2}L_{Pl}\cos\theta_1} = \frac{const}{p}\cdot\sqrt{\pi}(2m)^{\frac{1}{2}}\Gamma\left(\frac{3}{2}+\frac{1}{2}\right)p\left(m^2+\vec{p}^{\,2}\right)^{-1}\times \pi\frac{J_1\left(\sqrt{m^2+\vec{p}^{\,2}}\,L_{Pl}\right)}{\sqrt{m^2+\vec{p}^{\,2}}\,L_{Pl}}. \qquad (42)$$

Thus, after normalization, we have

$$D_s^Y(p) = \frac{1}{m^2 + \vec{p}^2} \cdot \frac{J_1\left(\sqrt{m^2 + \vec{p}^2}\, L_{Pl}\right)}{\sqrt{m^2 + \vec{p}^2}\, L_{Pl}} \qquad (43)$$

as it should be.

It is important to notice that the form of the Yukawa propagator in the momentum space has universal character independing on numbers of spacetime dimensions in the static limit:

$$\frac{1}{m^2 - \vec{p}^2 - i\epsilon} \Rightarrow \frac{1}{m^2 + \vec{p}^2} = const \int_0^\infty d\, r r^{D-2} \left(\frac{m}{r}\right)^{\frac{D-3}{2}} \times K_{\frac{D-3}{2}}(mr) \frac{J_{\frac{D-3}{2}}(pr)}{(pr)^{\frac{D-3}{2}}}, \qquad (44)$$

where we have used the following integral [3]:

$$\int_0^\infty d\, x x K_\nu(mx) J_\nu(px) = \frac{p^\nu}{m^\nu \left(m^2 + \vec{p}^2\right)}, \qquad (45)$$

and

$$D = 5,6, \dots, \qquad \vec{p}^2 = p_1^2 + p_2^2 + \cdots + p_{D-1}^2.$$

C. Calculation of Formulas in Any Spacetime Dimensions

a. Photon and graviton cases:

1) Let $D = 6$, then formula (31) takes the form:

$$D_6^Y = const \int_0^\infty d\, r \frac{r^4}{r^3} \int_0^{2\pi} d\, \varphi \int_0^\pi d\, \theta \cdot \sin\theta \times \int_0^\pi d\, \theta_1 \sin^2\theta_1 e^{ipr\cos\theta_1} \int_0^\pi d\, \theta_2 \sin^3\theta_2 e^{ipL_{Pl}\cos\theta_2},$$

where

$$U_C(r) \sim \frac{1}{r^3}.$$

Then

$$I_1 = \int_0^\pi d\, \theta_1 \sin\theta_1 \sqrt{1 - \cos^2\theta_1} e^{ipr\cos\theta_1} =$$

$$2\int_0^1 d\, x\sqrt{1 - x^2}\cos(prx) = 2 \cdot \frac{\pi}{2}\left(\frac{2}{pr}\right) \Gamma\left(1 + \frac{1}{2}\right) J_1(pr). \qquad (46)$$

So that

$$I_2 = \frac{1}{p}\int_0^\infty d\, r \cdot r \cdot \frac{1}{r} J_1(pr) = \frac{1}{p^2}$$

as it should be.

Thus

$$I_3 = \int_0^\pi d\, \theta_2 \sin^3\theta_2 e^{ipL_{Pl}\cos\theta_2} = 2\int_0^1 dx\, (1 - x^2)\cos(pL_{Pl}x) = \sqrt{\pi}\left(\frac{2}{pL_{Pl}}\right)^{3/2} \Gamma(2) J_{3/2}(pL_{Pl}). \qquad (47)$$

Finally, after normalization, we have

$$D_6^Y\left(\vec{p}^2\right) = \frac{1}{\vec{p}^2} \frac{J_{3/2}(pL_{Pl})}{(pL_{Pl})^{3/2}} \qquad (48)$$

2) Let $D = 7$, then we have similarity:

$$D_7^Y\left(\vec{p}^2\right) = const \int_0^\infty d\, r \frac{r^5}{r^4} \int_0^{2\pi} d\, \varphi \int_0^\pi d\, \theta \cdot \sin\theta \int_0^\pi d\, \theta_1 \sin^2\theta_1 \times \int_0^\pi d\, \theta_2 \sin^3\theta_2 e^{ipr\cos\theta_2} \int_0^\pi d\, \theta_3 \sin^4\theta_3 e^{ipL_{Pl}\cos\theta_3},$$

Where

$$I_4 = \int_0^\pi d\, \theta_2 \sin^3\theta_2 e^{ipr\cos\theta_2} = 2\int_0^1 d\, x(1 - x^2)\cos(prx)$$

$$= 2 \cdot \frac{\sqrt{\pi}}{2}\left(\frac{2}{pr}\right)^{3/2} \Gamma(2) J_{3/2}(pr),$$

$$I_5 = \frac{1}{p^{3/2}}\int_0^\infty d\, r \cdot r^{-1/2} J_{3/2}(pr) \Rightarrow \frac{1}{\vec{p}^2},$$

$$I_6 = \int_0^\pi d\, \theta_3 \sin^4\theta_3 e^{ipL_{Pl}\cos\theta_3} =$$

$$2\int_0^1 d\, x(1 - x^2)^{\frac{3}{2}}\cos(pL_{Pl}x) =$$

$$\sqrt{\pi}\left(\frac{2}{pL_{Pl}}\right)^2 \Gamma\left(\frac{5}{2}\right) J_2(pL_{Pl}).$$

Finally, we have as before

$$D_7^Y\left(\vec{p}^2\right) = \frac{1}{\vec{p}^2} \frac{J_2(pL_{Pl})}{(pL_{Pl})^2}. \qquad (49)$$

Similar calculations for any D -space lead to final result

$$D_D^Y(\vec{p}) = \frac{1}{\vec{p}^2} \frac{J_{\frac{D-3}{2}}(pL_{Pl})}{(pL_{Pl})^{\frac{D-3}{2}}}, \qquad (50)$$

where $D = 5,6 \dots$.

b. The Yukawa case

The Yukawa potential in 4-dimensional spacetime takes the well-known form:

$$U_4^Y(r) \sim \frac{1}{r} e^{-mr}. \qquad (51)$$

Then, $D = 4$:

$$D_4^Y = const \int_0^\infty d\, r \frac{r^2}{r} e^{-mr} \int_0^{2\pi} d\, \varphi \int_0^\pi d\, \theta \sin\theta e^{ipr\cos\theta},$$

where

$$I_7 = \int_0^{\pi} d\,\theta \sin\theta e^{iprcos\theta} = 2\int_0^1 d\,x\cos(prx) =$$
$$2\frac{\sqrt{\pi}}{2}\left(\frac{2}{pr}\right)^{1/2}\Gamma(1)J_{1/2}(pr)\,.$$

So that

$$I_8 = \frac{1}{\sqrt{p}}\int_0^{\infty} d\,r r^{1/2}J_{1/2}(pr) = \frac{1}{\sqrt{p}}\frac{(2p)^{1/2}\Gamma(1)}{\sqrt{\pi}\left(m^2+\vec{p}^{\,2}\right)},$$

therefore

$$D_4^Y(p) = \frac{1}{m^2+\vec{p}^{\,2}}.$$

In x-space, we have

$$U_4^Y(r) = const\int_0^{\infty} d\,p\frac{p^2}{m^2+p^2}\int_0^{2\pi} d\,\varphi\int_0^{\pi} d\,\theta\sin\theta e^{iprcos\theta},$$

$$I_8 = I_7 = \sqrt{\pi}\left(\frac{2}{pr}\right)^{1/2}\Gamma(1)J_{1/2}(pr).$$

Thus,

$$U_4^Y(r) = const\frac{1}{\sqrt{r}}\int_0^{\infty} d\,p\frac{p^2}{m^2+p^2}\frac{1}{\sqrt{p}}J_{1/2}(pr) =$$
$$\frac{C}{\sqrt{r}}\int_0^{\infty} d\,p\frac{p^{3/2}}{m^2+p^2}J_{1/2}(pr) = \sqrt{\frac{\pi}{2}}\frac{C}{r}e^{-mr},$$

where C is the normalization constant.

If $D = 5$, then

$$U_5^Y(r) = const\int_0^{\infty} d\,p\frac{p^3}{m^2+p^2}\int_0^{2\pi} d\,\varphi\int_0^{\pi} d\,\theta\sin\theta\times$$
$$\int_0^{\pi} d\,\theta_1\sin^2\theta_1 e^{iprcos\theta_1},$$

where

$$I_1 = \int_0^{\pi} d\,\theta_1\sin\theta_1\sqrt{1-\cos^2\theta_1}e^{iprcos\theta_1} = \sqrt{\pi}\left(\frac{2}{pr}\right)\Gamma\left(\frac{3}{2}\right)J_1(pr),$$

so that

$$U_5^Y(r) \sim \frac{1}{r}\int_0^{\infty} d\,p\frac{p^2}{m^2+p^2}J_1(pr) \sim \frac{K_1(mr)}{mr}. \qquad (52)$$

Therefore

$$D_5^Y(p) = const\cdot\int_0^{\infty} d\,r\frac{r^3}{r}K_1(mr)\times$$
$$\int_0^{2\pi} d\,\varphi\int_0^{\pi} d\,\theta\sin\theta\int_0^{\pi} d\,\theta_1\sin^2\theta_1 e^{iprcos\theta_1} = \frac{1}{m^2+\vec{p}^{\,2}}$$

as it should be.

Similar calculations for $U_6^Y(r)$ give

$$I_9 = 2\int_0^1 d\,x(1-x^2)\cos(prx) = \sqrt{\pi}\left(\frac{2}{pr}\right)^{3/2}\Gamma(2)J_{3/2}(pr),$$

and therefore

$$U_6^Y(r) \sim \frac{1}{r^{3/2}}\int_0^{\infty} d\,p\frac{p^{1+\frac{3}{2}}}{m^2+p^2}J_{3/2}(pr) \Rightarrow$$
$$const\cdot\frac{K_{3/2}(mr)}{(mr)^{3/2}}\cdot m^3. \qquad (53)$$

For the Yukawa potential case, we have used integrals (17) and (45).

Finally, for the Yukawa potential case for any D-dimensional spacetime, we have

$$U_D^Y(r) = const\cdot\left(\frac{m}{r}\right)^{\frac{D-3}{2}}K_{\frac{D-3}{2}}(mr) \qquad (54)$$

Now we want to calculate the Yukawa propagator for any D-dimensional spacetime. Thus, from (54) one gets:

If $D = 5$, then

1.

$$D_5^Y(p) = const\int_0^{\infty} d\,r\frac{r^3}{r}K_1(mr)\int_0^{2\pi} d\,\varphi\times$$
$$\int_0^{\pi} d\,\theta\cdot\sin\theta e^{ip'L_{Pl}\cos\theta}\int_0^{\pi} d\,\theta_1\sin^2\theta_1 e^{ip'rcos\theta_1}. \qquad (55)$$

where

$$I_{10} = \int_0^{\pi} d\,\theta\sin\theta e^{ip'L_{Pl}cos\theta} = 2\int_0^1 d\,x\cos(p'L_{Pl}x)$$
$$= 2\cdot\frac{\sqrt{\pi}}{2}\left(\frac{2}{p'L_{Pl}}\right)^{1/2}\Gamma(1)J_{1/2}(p'L_{Pl})$$

and

$$I_{11} = \int_0^{\pi} d\,\theta_1\sin^3\theta_1 e^{iprcos\theta_1} =$$
$$2\int_0^1 d\,x\sqrt{1-x^2}\cos(prx)$$
$$= 2\frac{\sqrt{\pi}}{2}\left(\frac{2}{pr}\right)\Gamma\left(\frac{3}{2}\right)J_1(pr).$$

Therefore

$$D_s^Y(p) = const\frac{J_{1/2}(p'L_{Pl})}{\sqrt{p'L_{Pl}}}\frac{1}{p}\int_0^{\infty} d\,r rK_1(mr)J_1(pr),$$

here

$$\int_0^{\infty} d\,r rK_1(mr)J_1(pr) = \frac{p}{m\left(m^2+\vec{p}^{\,2}\right)}.$$

So that

$$D_5^Y(p) = \frac{1}{m^2+\vec{p}^{\,2}}\frac{J_{1/2}(p'L_{Pl})}{\sqrt{p'L_{Pl}}}, \qquad (56)$$

where

$$p' = \sqrt{m^2 + p^2}.$$

2. The case $D = 6$ reads

$$D_6^Y(p) = const \int_0^\infty d\,r \frac{r^4}{r^{3/2}} K_{3/2}(mr) \int_0^{2\pi} d\,\varphi \times$$

$$\int_0^\pi d\,\theta \cdot \sin\theta \int_0^\pi d\,\theta_1 \sin^2\theta_1 e^{ip'L_{Pl}\cos\theta_1} \int_0^\pi d\,\theta_2 \sin^3\theta_2 e^{iprcos\theta_2}.$$

Here we use integral forms I_1 (46) and I_3 (47).

Then

$$D_6^Y(p) = const \cdot \frac{J_1(p'L_{Pl})}{p'L_{Pl}} \cdot \frac{1}{p^{3/2}} \times$$

$$\int_0^\infty d\,r \frac{r^4}{r^{3/2}} \cdot \frac{1}{r^{3/2}} K_{3/2}(mr) J_{3/2}(pr).$$

Here

$$\int_0^\infty d\,r r K_{3/2}(mr) J_{3/2}(pr) = \frac{p^{3/2}}{m^{3/2}\left(m^2 + \vec{p}^2\right)}.$$

Thus, after normalization we have

$$D_6^Y(p) = \frac{1}{m^2 + \vec{p}^2} \times \frac{J_1(p'L_{Pl})}{p'L_{Pl}} \qquad (57)$$

Finally, analogous calculations for any D-space read universe formula for the Yukawa propagator in the static limit:

$$D_D^Y(p) = \frac{1}{m^2 + \vec{p}^2} \frac{J_{\frac{D-4}{2}}(p'L_{Pl})}{(p'L_{Pl})^{\frac{D-4}{2}}} \qquad (58)$$

where $p' = \sqrt{m^2 + \vec{p}^2}$.

D. *Euclidean Propagators in D-dimensions*

a. *Local Euclidean Propagators for Photons and Gravitons*

Local Euclidean propagators are given by the universe formula for any D-dimensions:

$$D_D^{Y,g} = \frac{1}{(2\pi)^D} \Omega(D-1) \int_0^\infty d\,p \frac{p^{D-1}}{p^2} \times \int_0^1 d\,\theta \sin^{D-2}\theta e^{iprcos\theta}, \qquad (59)$$

where $\Omega(D-1)$ is the area of a unit sphere in the D-dimensional volume, which is given by the universal formula

$$\Omega(D-1) = \frac{2\pi^{(D-1)/2}}{\Gamma\left(\frac{D-1}{2}\right)}. \qquad (60)$$

To get this formula we use the following integral formulas

$$\int_0^\pi d\,\theta (\sin\theta)^k = \sqrt{\pi} \frac{\Gamma\left(\frac{1+k}{2}\right)}{\Gamma\left(\frac{2+k}{2}\right)}. \qquad (61)$$

In particular, if $D = 3$, then $\Omega(2) = \frac{2\pi^{(3-1)/2}}{\Gamma\left(\frac{3-1}{2}\right)} = 2\pi$. For other cases:

$$\begin{aligned} \Omega(3) &= 4\pi \quad \text{if} \quad D = 4 \\ \Omega(4) &= 2\pi^2 \quad \text{if} \quad D = 5 \\ \Omega(5) &= \frac{8}{3}\pi^2 \quad \text{if} \quad D = 6 \end{aligned}$$

and so on.

In the integral formula (59) we have

$$I_1 = \int_0^\pi d\,\theta \sin^{D-2}\theta e^{iprcos\theta} = 2\int_0^1 d\,x (1-x^2)^{\frac{(D-3)}{2}} \cdot cosprx$$

$$= 2\frac{\sqrt{\pi}}{2}\left(\frac{2}{pr}\right)^{\frac{D-2}{2}} \Gamma\left(\frac{D-2}{2} + \frac{1}{2}\right) J_{\frac{D-2}{2}}(pr). \qquad (62)$$

Now we use other integral formula:

$$I_2^Y = \int_0^\infty d\,p \frac{p^{D-1}}{p^2 p^{\frac{D-2}{2}}} J_{\frac{D-2}{2}}(pr) = 2^{\frac{D-4}{2}} r^{-1-\frac{D-4}{2}} \Gamma\left(\frac{D-2}{2}\right) \qquad (63)$$

Finally, we have universal formula for local photon and graviton fields:

$$D_D^{Y,g}(r) = \sqrt{\pi} \frac{\Omega(D-1)}{(2\pi)^D} 2^{D-3} \cdot \Gamma\left(\frac{D-1}{2}\right) \Gamma\left(\frac{D-2}{2}\right) r^{2-D}, \qquad (64)$$

where $D = 4,5,6, \dots$ are numbers of D-dimensional spacetime. This formula is the D-dimensional local photon and graviton propagators in x-space, where $r = \sqrt{x_1^2 + x_2^2 + \cdots + x_D^2}, x_1 = ix_0$.

b. *Local Euclidean propagator for a scalar particle*

Extersion above formulas for a scalar particle are obviously. Indeed

$$D_D^Y(r) = \frac{1}{(2\pi)^D} \Omega(D-1) \int_0^\infty d\,p \frac{p^{D-1}}{m^2 + p^2} \times \int_0^1 d\,\theta \sin^{D-2}\theta e^{iprcos\theta}. \qquad (65)$$

In this case, integral (62) is the same and while integral (63) takes the form

$$I_2^Y = \int_0^\infty d\,p \frac{p^{D/2}}{m^2 + p^2} J_{\frac{D-2}{2}}(pr) = m^{\frac{D-4}{2}} K_{\frac{D-2}{2}}(mr). \qquad (66)$$

Therefore, one gets:

$$D_D^Y(r) = \sqrt{\pi}\frac{\Omega(D-1)}{(2\pi)^D}\Gamma\left(\frac{D-1}{2}\right)\left(\frac{2m}{r}\right)^{\frac{D-2}{2}} K_{\frac{D-2}{2}}(mr) \qquad (67)$$

c. Nonlocal Euclidean Propagators for Photons and Gravitons

For this case, the following universal formulas are valid:

$$D_{L_{Pl}}^{\gamma,g}(r) = \sqrt{\pi}\frac{\Omega(D-1)}{(2\pi)^D}\left(\frac{2}{r}\right)^{\frac{D-2}{2}}\Gamma\left(\frac{D-1}{2}\right)\frac{1}{(L_{Pl})^{\frac{D-3}{2}}}\times \int_0^\infty dp\frac{p^{D-1}}{p^2\cdot p^{\frac{D-2}{2}}}\cdot\frac{J_{\frac{D-3}{2}}(pL_{Pl})}{(p)^{\frac{D-3}{2}}}J_{\frac{D-2}{2}}(pr). \qquad (68)$$

Now we want to calculate the following integral:

$$I_3 = \int_0^\infty dp\, p^{-1/2} J_\nu(pr)J_{\nu-1/2}(pL_{Pl}), \qquad (69)$$

where $\nu = (D-2)/2$. By using the integral form [3], one gets:

$$I_3 = \begin{cases} \frac{1}{\sqrt{2\pi}}\frac{1}{\sqrt{L_{Pl}}}\left(\frac{r}{L_{Pl}}\right)^{\frac{D-2}{2}}\frac{\Gamma\left(\frac{D-2}{2}\right)}{\Gamma\left(\frac{D}{2}\right)}F\left(\frac{D-2}{2},\frac{1}{2};D;\frac{r^2}{L_{Pl}^2}\right), \\ \text{where} 0 < r < L_{Pl} \\ \frac{1}{\sqrt{2L_{Pl}}}\frac{\Gamma\left(\frac{D-2}{2}\right)}{\Gamma\left(\frac{D-1}{2}\right)}, \text{ for } r = L_{Pl} \\ \frac{1}{\sqrt{2r}}\left(\frac{L_{Pl}}{r}\right)^{\frac{D-3}{2}}\frac{\Gamma\left(\frac{D-2}{2}\right)}{\Gamma\left(\frac{D-1}{2}\right)}F\left(\frac{D-2}{2},\frac{1}{2};D-1;\frac{r^2}{L_{Pl}^2}\right) \text{for } 0 < L_{Pl} < r, \end{cases} \qquad (70)$$

where $F(\alpha,\beta,\gamma,x)$ is the hypergeometric function.

d. Nonlocal Euclidean Propagator for Scalar Particle with Mass $\boldsymbol{m}$

A Similar formula with respect to (68) holds:

$$D_{L_{Pl}}^{Y}(r) = \sqrt{\pi}\frac{\Omega(D-1)}{(2\pi)^D}\left(\frac{2}{r}\right)^{\frac{D-2}{2}}\frac{\Gamma\left(\frac{D-1}{2}\right)}{(L_{Pl})^{\frac{D-4}{2}}}\times I_4(r), \qquad (71)$$

where

$$I_4(r) = \int_0^\infty dp\frac{p^2}{m^2+p^2}J_\nu(pr)J_{\nu-1}(pL_{Pl}). \qquad (72)$$

Here $\nu = (D-2)/2$. We use the integral formula [3], one gets

$$I_4(r) = mI_{\frac{D-4}{2}}(mL_{Pl})K_{\frac{D-2}{2}}(mr). \qquad (73)$$

Finally, we have

$$D_{L_{Pl}}^{Y}(r) = \sqrt{\pi}\frac{\Omega(D-1)}{(2\pi)^D}(mL_{Pl})\left(\frac{2}{rL_{Pl}}\right)^{\frac{D-2}{2}}\times I_{\frac{D-4}{2}}(mL_{Pl})K_{\frac{D-2}{2}}(mr). \qquad (74)$$

Notice that physical formulas in any D-dimensional spacetime have universal and beautiful properties.

E. Hypothesis of the Planck Regularization Procedure

Thus, in our approach force carrying photon, graviton and $W^\pm, Z^0$ -type spin 1 vectors, and also scalar particle's propagators in any D-dimensions of spacetime with fundamental length named the Planck length take the forms:

$$D_{\mu\nu}^{\gamma}(x_D) = \frac{g^{\mu\nu}}{(2\pi)^D i}\int d^D p e^{ipx}\frac{V(-p_D^2L_{Pl}^2)}{-p_D^2 - i\epsilon}, \qquad (75)$$

$$D_{\mu\nu,\rho\delta}^{g}(x_D) = \frac{\Delta_{\mu\nu,\rho\delta}}{(2\pi)^D i}\int d^D p e^{ipx}\frac{V(-p_D^2L_{Pl}^2)}{-p_D^2 - i\epsilon}, \qquad (76)$$

for photon and graviton fields;

$$D^m(x_D) = \frac{1}{(2\pi)^D i}\int d^D p e^{ipx}\frac{V_m(-p_D^2L_{Pl}^2)}{m^2 - p_D^2 - i\epsilon}, \qquad (77)$$

for scalar particles;

$$D_{\mu\nu}^m(x_D) = \frac{1}{(2\pi)^D i}\int d^D p e^{ipx}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}\right)\frac{V_m(-p_D^2L_{Pl}^2)}{m^2 - p_D^2 - i\epsilon}, \qquad (78)$$

for spin one massive vector $W^\pm, Z^0$-bosons, which are carrying electro-weak interactions. Here, formfactors

$$V(-p_D^2L_{Pl}^2) = \frac{1}{2^\lambda}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi\frac{\left(-\frac{1}{4}p_D^2L_{Pl}^2\right)^\xi}{\sin\pi\xi\Gamma(1+\xi)\Gamma(\lambda+\xi+1)}, \qquad (79)$$

$$V_m(-p_D^2L_{Pl}^2) = \frac{1}{2^{\lambda_i}}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi\frac{\left[\frac{(m^2-p_D^2)L_{Pl}^2}{4}\right]^\xi}{\sin\pi\xi\Gamma(1+\xi)\Gamma(\lambda_1+\xi+1)} \qquad (80)$$

satisfy the Mellin representations. Here

$$ipx = ip_0x_0 - ip_1x_1 - \cdots - ip_{D-1}x_{D-1},$$
$$p_D^2 = p_0^2 - p_2^2 - \cdots - p_{D-1}^2, \qquad 0 < \beta < 1,$$
$$\lambda = \frac{D-3}{2}, \qquad \lambda_1 = \frac{D-4}{2}, \qquad D = 5,6,7,\dots,$$

respectively.

Notice that due to formfactors (79) and (80) which are analytic functions on the left hand complex plane and there are no poles there, all Feynman diagrams (expect vacuum polarization ones which are regularized by using D-dimensional procedure [7]) for electromagnetic, electro-weak and gravitational interactions between elementary particles, and also for self-interactions of scalar particles are finite and free from ultraviolet divergences. Thus, we arrive at the nonlocal quantum field theory developed by Dubna group due to Efimov [8] and Blokhintsev [9].

In this nonlocal theory, force carrying boson fields, like photons, gravitons, $W^{\pm}, Z^0$-bosons and so on became nonlocal and their causal or Green functions in the S-matrix are determined by above formulas (75)-(78) with form factors (79) and (80).

Thus, vacuum expectations of T-product of these fields are given by formulas:

$$\begin{gathered}\langle 0|T\{A_\mu^{L_{Pl}}(x)A_\nu^{L_{Pl}}(y)\}|0\rangle = \frac{1}{i}D_{\mu\nu}^L(x-y),\\ \langle 0|T\{G_{\mu\nu}^L(x)G_{\rho\delta}^L(y)\}|0\rangle = \frac{1}{i}\Delta_{\mu\nu,\rho\delta}D_0^L(x-y),\\ \langle 0|T\{W_\mu^L(x)W_\nu^L(y)\}|0\rangle = \frac{1}{i}D_{m,\mu\nu}^L(x-y), (L=L_{Pl})\end{gathered} \qquad (81)$$

and so on.

Meanwhile, as usual all fermionic fields are local and those causal or Green functions do not changed and are determined by usual expressions in the local theory. Indeed, if fermionic fields, for example electron ones are changed then the unitarity and gauge invariance properties of S-matrix are broken (for example, see Section III).

Free and interaction Lagrangians in D-dimensional spacetime are constructed by means of fields $\Phi^i(x_D^\nu), \Psi(x_D^\nu)$ and those differentials $\partial\Phi^i(x_D^\mu)/\partial(x_D^\nu)$, $\partial\Psi(x_D^\nu)/\partial(x_D^\mu)$ and their the general form reads:

$$L_{free}\left(\Phi^i(x_D^\mu), \partial\Phi^i/\partial x_D^\rho\right) + L_{free}\left(\Psi(x_D^\mu), \partial\Psi/\partial x_D^\lambda\right) + L_{in}\left(\Psi(x_D^\mu), \Phi^i(x_D^\nu)\right).$$

Then S-matrix for interaction between $\Phi^i(x_D^\mu)$ and $\Psi(x_D^\nu)$-fields is given by the form

$$S = T\exp\{i\int d^D x g(x_D) L_{in}(\dots)\}, \qquad (82)$$

where we insert an adiabatic switching function $g(x_D)$ that turns the interaction on and off by hand in some domain Γ (see Figure 1):

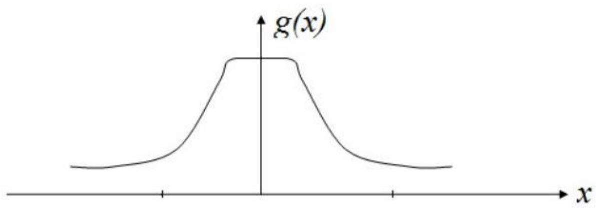

Fig. 1. *The adiabatic function* $g(x_D)$

By means of $g(x_D)$-function the causality condition is formulated due to Bogolubov and Shirkov [10]. Thus, we introduce an operation of "switching on" and "switching off" the interaction. Instead of the coupling constant g, we introduce the function $g(x_D)$ on the interval $[0, g]$, which characterize the intensity of "switching on" the interaction. Let $g_1(x_D)$ be different from zero in some region $\Gamma \subset R^D$ and $g_2(x_D)$ in $\Gamma_2 \subset R^D$. Then the S-matrix of the theory satisfies the **microcausality condition**, if

$$S[g_1 + g_2] = S[g_2]S[g_1] \qquad (83)$$

if $\Gamma_2 \gtrsim \Gamma_1$, i.e. if and only if all the points of the region Γ_2 belong to the future cone (or to the space-like region) with respect to all points of the region Γ_1. The condition (83) can be written in the differential form:

$$R(x,y) = \frac{\delta}{\delta g(x)}\left(\frac{\delta S}{\delta g(y)}S^+\right) = 0 \qquad (84)$$

for $x \leq y$. This relation represents a formulation of causality in the differential form [10].

Roughly speaking our approach to elimination of ultraviolet divergences in S-matrix by using concept of the fundamental length due to high-dimensional space-time is called gravitational effect regularization of the quantum field theory, because the Newtonian or gravitational constant G is involved in the Planck length's expression due to the formula (13).

Notice that the propagators of gluons and those T-products or Green functions in quantum chromodynamics are also defined by the similar formulas (75)-(78) and (81). In conclusion, we see that all these force carrying bosons fields $B(x_D)$ in any D-dimensional space-time became nonlocal and are given by the generalized operator functions $M(\square L_{Pl}^2)$ or $M_m((\square - m^2)L_{Pl}^2)$ depending on mass value:

$$B_{local}(x) \Rightarrow B_{nonlocal}^{L_{Pl}}(x_D) = M(\square L_{Pl}^2)B_{local}(x_D) \qquad (85)$$

where

$$M(\square L_{Pl}^2) = \left\{\frac{J_{\frac{D-2}{2}}(\square L_{Pl}^2)}{(\square L_{Pl}^2)^{\frac{D-3}{2}}}\right\}^{1/2} \qquad (86)$$

for massless boson fields, and

$$M_m((\square - m^2)L_{Pl}^2) = \left\{\frac{J_{\frac{D-4}{2}}((\square - m^2)L_{Pl}^2)}{((\square - m^2))^{\frac{D-3}{2}}}\right\}^{1/2} \qquad (87)$$

for massive bosons, where

$$\square = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \dots + \frac{\partial^2}{\partial x_{D-1}^2}. \qquad (88)$$

III. Nonlocal Quantum Electrodynamics

A. *Introduction*

In previous chapter we observed that an origin of the divergence problem in quantum electrodynamics is associated with a singularity of classical electrostatic field. By this reason in order to avoid singularity in the Coulomb potential we have considered D-dimensional spacetime and obtained its finite modification with using the concept of the fundamental length. A modification of the Coulomb potential in D-dimensional spacetime leads to the charge of the photon propagator due to the formula (75). Aim of this chapter is to construct finite nonlocal quantum electrodynamics. The beautiful quantum electrodynamics (QED) developed by many physicists of the 20th Century [11-15] has been played a vital role in the construction of the finite and gauge-invariant so-called standard model [16-17] of the particle physics.

The modification of the Coulomb potential

$$U_D^C(r_D) \sim \frac{1}{r_D^{D-3}} \qquad (89)$$

gives rise to the nonlocal photon propagator in D-dimensional spacetime (75).

As an example, here we study Feynman diagrams in nonlocal quantum electrodynamics (NQED) in which the photon propagator is changed and spinor propagator does not modified because of conservation of electric charge which is broken for this case. In the language of Feynman diagrams if we change spinor propagator then the Ward-Takahashi identity does not valid. For simplicity of calculation purpose, we consider 4-dimensional spacetime which is embodied into 5-dimensional one and therefore we use the particular formfactor $J_1(pL_{Pl})/(pL_{Pl})$ for the photon propagator in the momentum space:

$$D^\gamma(p) = \frac{V(p^2L_{Pl}^2)}{-p^2 - i\epsilon}, \qquad (90)$$

where

$$V(p^2L_{Pl}^2) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\,\xi\, \frac{v(\xi)}{\sin\pi\xi}(p^2L_{Pl}^2)^\xi \qquad (91)$$

$$v(\xi) = \frac{1}{2}2^{-2\xi}\frac{1}{\Gamma(1+\xi)\Gamma(2+\xi)}, \quad 0<\beta<1. \qquad (92)$$

Moreover, from formulas (68) and (70) we see that photon propagator $D_{\mu\nu}^\gamma(x)$ at the point $x=0$ is finite, and therefore, in principle, one can calculate vacuum fluctuation diagrams shown in Figure 2

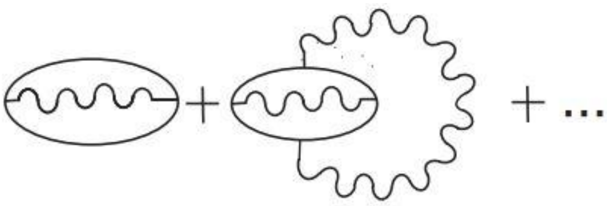

Fig. 2. *Primitive Feynman diagrams for vacuum fluctuation*

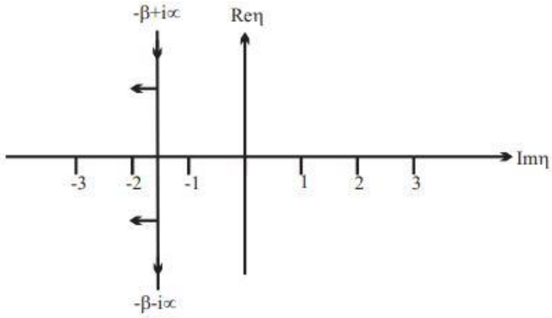

Fig.3. Integration contour in the formula (91)

Finally, we indicate one important consequence of the photon propagator (90) with the form - factor (91). If we want to calculate high order divergence integrals over the internal momentum variable, like

$$\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\,\xi\,\frac{v(\xi)}{\sin\pi\xi}\int d^4p\,\frac{[p^{2\nu}]^\xi}{[p^2+A]^\lambda}$$

for any order of ν, then we can move integration contour in Figure 3 to the left through points $\xi = -1,-2,-3,\ldots$, in desired order, since in such type of integrals there are no poles at these points. After integration result we can again move integration contour to the right to calculate residues at the points $\xi = -3,-2,-1,\ldots$ so on. This procedure of analytic continuation over complex variable ξ plays a vital role in regularization scheme.

Lagrangian functions of the nonlocal quantum electrodynamics arisen from the modification of the Coulomb potential in D-dimensional space time with propagator (75) have similar structures as the local theory [10].

$$\begin{aligned} L(x) = \; & e{:}\overline{\psi}(x)\hat{A}(l,x)\psi(x){:} + e(Z_1-1){:}\overline{\psi}(x)\hat{A}(l,x)\psi(x){:} \\ & -\delta m{:}\,\bar{\psi}(x)\psi(x){:} + (Z_2-1){:}\,\bar{\psi}(x)\big(i\hat{\partial}-m\big)\psi(x){:} \\ & (Z_3-1)\frac{1}{4}{:}F_{\mu\nu}(x)F^{\mu\nu}(x){:}, \end{aligned} \qquad (93)$$

whe $L = L_{Pl}, x_\mu = x_0, \vec{x}$; $\hat{A}(L,x) = A_\mu(L,x)\gamma^\mu, \hat{\partial} = \gamma^\mu \frac{\partial}{\partial x_\mu}$, $A_\mu(L,x)$ is the nonlocal photon field defined by the formula (85) with the generalized function (86).

Only in our case of the nonlocal theory, renormalization constants $Z_1, Z_2, Z_3, \delta m$ are finite and moreover $Z_1 = Z_2$ due to the Ward-Takahashi identity. Here "Chronological" pairing (or T-product) of the fermionic field operators of electrons has the usual local form:

$$\begin{aligned} S(x-y) &= \langle 0|T\big[\psi(x)\overline{\psi}(y)\big]|0\rangle \\ &= \frac{1}{(2\pi)^4}\frac{1}{i}\int d^4p\,\frac{e^{-ip(x-y)}}{m-\hat{p}-i\varepsilon} \end{aligned} \qquad (94)$$

while "causal" function of the nonlocal electromagnetic field $A_\mu(L,x)$ in (93) takes the form

$$\begin{aligned} & D_{\mu\nu}^L(x-y) = g_{\mu\nu}D^L(x-y) = \\ & -\frac{g_{\mu\nu}}{(2\pi)^4 i}\int d^4p\,\frac{V(-p^2L_{Pl}^2)}{-p^2-i\epsilon}e^{-ip(x-y)} \end{aligned} \qquad (95)$$

due to the formula (90). Here $V(-p^2 L_{Pl}^2)$ is given by the formula (91) with using (92). In calculations of Feynman diagrams, we use the Feynman parametric formula

$$\frac{1}{a^{n_1} b^{n_2}} = \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)} \int_0^1 d\, x x^{n_1-1}(1-x)^{n_2-1} \times \frac{1}{[ax+b(1-x)]^{n_1+n_2}} \tag{96}$$

Notice that free Lagrangian of the electromagnetic field in (93) is constructed by using usual local tensor field:

$$F_{\mu\nu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x).$$

Due to presence of the nonlocal photon propagator (95) in our theory all its matrix elements corresponding to any Feynman diagrams are as usually calculated by using the high-dimensional regularization method of ’t Hooft and Veltman [7].

Now we would like to calculate some primitive Feynman diagrams in NQED arisen from influence gravity from four-dimensional spacetime which is embodied into five-dimensions.

B. The Electron Self - Energy in NQED

The complete electron propagator in NQED is given by the sum

$$[-i(2\pi)^{-4}S_l(p)] = [-i(2\pi)^{-4}S(p)] + [i(2\pi)^{-4}S(p)][i(2\pi)^4\Sigma_l(p)] \times [-i(2\pi)^{-4}S(p)] + \cdots \tag{97}$$

where

$$S(p) = \frac{m+p}{m^2-p^2-i\varepsilon}, \quad L_{Pl} = l.$$

The sum is trivial and gives

$$S_L(p) = [m - p - \Sigma_l - i\varepsilon]^{-1}.$$

In lowest order there is a one - loop contribution to Σ_l, given by in Figure 4: and corresponding expression takes the form:

$$-i{:}\overline{\psi}(x)\Sigma_l(x-y)\psi(y){:},$$

where

$$\Sigma_l(x-y) = -ie^2\gamma_\mu S(x-y)\gamma_\mu D^l(x-y). \tag{98}$$

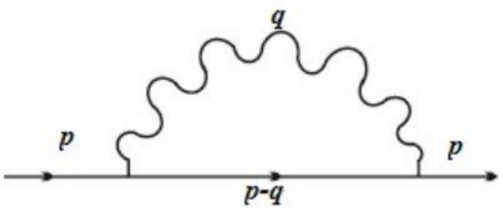

Fig.4. *Diagram of Self - energy of a electron in NQED*

Passing to the momentum representation and making us of our regularization procedure which is connected introduction of the fundamental length named the Planck one that allows us to go to the Euclidean metric by using $k_0 \to \exp(i\pi/2)k_4$, one gets ($L = L_{Pl}$):

$$\Sigma_L(q) = \frac{e^2}{(2\pi)^4}\int d^4k_E \frac{V(k_E^2 L^2)}{k_E^2} \times \gamma_\mu^{(E)} \frac{m - \hat{p}_E + \hat{k}_E}{m^2 + (p_E - k_E)^2}\gamma_\mu^{(E)} \tag{99}$$

Here $p_E = \left(-ip_0, \vec{p}\right), \gamma^{(E)} = \left(-i\gamma_0, \vec{\gamma}\right)$ and $k_E = \left(k_4, \vec{k}\right)$.

Taking into account the Mellin representation (91) for the form - factor $V(k_E^2 l^2)$ and after some calculations, we have [8]:

$$\tilde{\Sigma}_l(p) = \frac{e^2}{8\pi}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\,\xi \frac{1}{\sin^2\pi\xi}\frac{v(\xi)(m^2 l^2)^\xi}{\Gamma(1+\xi)}F(\xi,p)$$

where

$$v(\xi) = \frac{1}{\Gamma(2+2\xi)}\frac{1}{\Gamma(1+\xi)}\cdot\frac{1}{2}2^{-2\xi},$$

and

$$F(\xi,p) = \frac{1}{\Gamma(1-\xi)}\int_0^1 d\,u\left(\frac{1-u}{u}\right)^\xi\left(1-\frac{p^2}{m^2}u\right)^\xi \times (2m - pu) \tag{101}$$

is a regular function in the half - plane $Re\,\xi > -1$.

Assuming the value $m^2 l^2 = m^2 L_{Pl}^2$ is to be small, one can obtain:

$$\begin{aligned}\tilde{\Sigma}_l(p) = {} & \frac{e^2}{8\pi^2}\int_0^1 d\,u(2m-\hat{p}u)\ln\left(1-\frac{p^2}{m^2}u\right) - \\ & -\frac{e^2}{16\,\pi^2}\Big[\left(3\ln\frac{1}{m^2l^2} + 3v'(0) + 3\psi(1) + 1\right) \\ & +4m^2L^2 v(1)\left(\ln\frac{1}{m^2l^2} - \frac{v'(1)}{v(1)} - \frac{5}{12}\frac{p^2}{m^2}\right)\Big] - \\ & -\frac{e^2}{16\,\pi^2}(m-\hat{p})\left[\left(\ln\frac{1}{m^2l^2}\, v'(0)+1\right) - m^2l^2v(1)\frac{p^2}{3m^2}\right].\end{aligned} \tag{102}$$

where $\psi(1)$ is the psi-function, which is connected with the Euler numbers C by the equality:

$$\psi(1) = -C = -0.57721566490 \ldots.$$

We see that all calculations are finite due to introduction of the Planck length into the theory.

C. Vertex Function and the Anomalous Magnetic Moment of Leptons in NQED

In the momentum space and in the Euclidean metric, the vertex function takes the form [Figure 5]:

$$\tilde{\Gamma}_\mu^l(p_1,p) = -\frac{e^2}{(2\pi)^4}\int d^4k_E \frac{V((p_E-k_E)^2 l^2)}{(p_E-k_E)^2}\gamma_\nu \times \times \frac{m - \hat{k}_E - \hat{q}_E}{m^2 + (k_E+p_E)^2}\gamma_\mu \frac{m-\hat{k}_E}{m^2+k_E^2}\gamma_\nu \tag{103}$$

Again passing to the Minkowski metric and using the generalized Feynman parameterization formula (96) one gets:

$$\tilde{\Gamma}^{l}_{\mu}(p_1;p) = -\frac{e^2}{8\pi}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\,\xi\,\frac{v(\xi)}{(\sin\pi\xi)^2}\frac{(m^2l^2)^{\xi}}{\Gamma(1+\xi)}F_{\mu}(\xi;p_1,p) \qquad (104)$$

where

$$F_{\mu}(\xi;p_1,p) = \gamma_{\mu}F_1(\xi;p_1,p) + F_{2\mu}(\xi;p_1,p).$$

Here

$$\begin{aligned} F_1(\xi;p_1,p) &= \frac{1}{\Gamma(1-\xi)}\int_0^1\int_0^1\int_0^1 d\,\alpha d\beta d\gamma\delta(1-\alpha-\beta-\gamma) \\ &\quad\times \alpha^{-\xi}Q^{\xi}, \\ F_{2\mu}(\xi;p_1,p) &= \frac{1}{\Gamma(-\xi)}\int_0^1\int_0^1\int_0^1 d\,\alpha d\beta d\gamma\delta(1-\alpha-\beta-\gamma) \\ &\quad\times \alpha^{-\xi}Q^{\xi-1}\frac{1}{m^2}[m^2\gamma_{\nu} - 2mq_{\mu} + 4m(\beta q_{\mu} - \alpha p_{\mu}) + \\ &\quad + (\alpha\hat{p} - \beta\hat{q})\gamma_{\mu}\hat{q} + (\alpha\hat{p} - \beta\hat{q})\gamma_{\mu}(\alpha\hat{p} - \beta\hat{q})], \end{aligned} \qquad (105)$$

and

$$Q = \beta + \gamma - \alpha\gamma\frac{p^2}{m^2} - \beta\gamma\frac{q^2}{m^2} - \alpha\beta\frac{(p+q)^2}{m^2}.$$

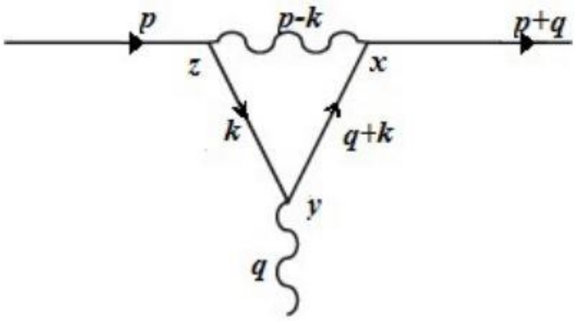

Fig.5. *Vertex function in NQED*

Let us calculate the vertex function (104) for two cases: first, when $q = 0$ and p has an arbitrary value; second, when q is an arbitrary quantity and p, p_1 are situated on the m - mass shell.

In the first case, assuming $q = 0$ in the formula (105) and after some standard calculations [8], one gets

$$\begin{aligned} F_{\mu}(\xi;p_1,p) &= \frac{1}{\Gamma(1-\xi)}\int_0^1 d\,u\left(\frac{1-u}{u}\right)^{\xi}\left(1 - u\frac{p^2}{m^2}\right)^{\xi}\times \\ &\quad\times\left[u\,\gamma_{\mu} + \frac{2\xi\,u\,p_{\mu}(2m - u\,\hat{p})}{m^2 - u\,p^2}\right]. \end{aligned} \qquad (106)$$

Comparing this formula with the expression (101) for the self-energy of the electron, it is easily seen that

$$F_{\mu}(\xi;p,p) = -\frac{\partial}{\partial p_{\mu}}F(\xi;p) \qquad (107)$$

From this identity, we can obtain a very important conclusion. In the nonlocal theory of quantum electrodynamics constructed by using the concept of influence of gravity through introduction of the fundamental length named the Planck one in 4-dimensional spacetime which is embodied into 5-dimensional one, the Ward - Takahashi identity is valid

$$\tilde{\Gamma}^{l}_{\mu}(p,p) = -\frac{\partial}{\partial p_{\mu}}\tilde{\Sigma}_l(p) \qquad (108)$$

This is one of consequences of the gauge invariance, i.e., conservation of the electric charge in NQED.

In the second case, one can put

$$\bar{u}(\vec{p}_1)\Gamma^{l}_{\mu}(p_1,p)u(\vec{p}) = \bar{u}(\vec{p}_1)\Lambda_{\mu}(q)u(\vec{p}) \qquad (109)$$

where $\bar{u}(\vec{p}_1)$ and $u(\vec{p})$ are solutions of the Dirac equation

$$(p-m)u(\vec{p}) = 0, \bar{u}(\vec{p}_1)(p_1 - m) = 0.$$

Substituting the vertex function (104) into (109) and after some transformations, we have

$$\bar{u}(\vec{p}_1)F_{\mu}(\xi;p_1,p)u(\vec{p}) = \bar{u}(\vec{p}_1)\Lambda_{\mu}(\xi;q)u(\vec{p}) \qquad (110)$$

Here

$$\Lambda_{\mu}(\xi;q) = \gamma_{\mu}f_1(\xi;q^2) + \frac{i}{2m}\sigma_{\mu\nu}q_{\nu}f_2(\xi;q^2),$$

$$\sigma_{\mu\nu} = \frac{1}{2i}(\gamma_{\mu}\gamma_{\nu} - \gamma_{\nu}\gamma_{\mu}),$$

$$\begin{aligned} f_i(\xi;q^2) &= \frac{1}{\Gamma(1-\xi)}\int_0^1\int_0^1\int_0^1 d\,\alpha d\beta d\gamma\delta(1-\alpha-\beta-\gamma) \\ &\quad\times\alpha^{-\xi}M^{\xi-1}g_i(\alpha,\beta,\gamma,q^2), \end{aligned}$$

$$\begin{aligned} M &= \varepsilon\alpha + (1-\alpha)^2 - \beta\gamma\frac{q^2}{m^2}, \\ g_1(\alpha,\beta,\gamma,q^2) &= [(1-\alpha)^2(1-\xi) + 2\alpha\xi] - \\ &\quad - [\beta\gamma + \xi(\alpha+\beta)(\alpha+\gamma)]\frac{q^2}{m^2}, \\ g_2(\alpha,\beta,\gamma,q^2) &= 2\alpha(1-\alpha)\xi\,. \end{aligned} \qquad (111)$$

To avoid infrared divergences in the vertex function we have introduced here the parameter $\varepsilon = \mu^2_{ph}/m^2$, taking into account the "mass" of the photon. Finally, one gets

$$\Lambda_{\mu}(q) = \gamma_{\mu}F_1(q^2) + \frac{i}{2m}\sigma_{\mu\nu}q_{\nu}F_2(q^2), \qquad (112)$$

where

$$F_j(q^2) = -\frac{e^2}{8\pi}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}\frac{d\xi}{(\sin\pi\xi)^2}\frac{v(\xi)}{\Gamma(1+\xi)}(m^2l^2)^{\xi}f_j(\xi,q^2) \qquad (113)$$

It is easy to verify that the vertex function $\Lambda_{\mu}(q)$ satisfies the gauge invariant condition:

$$q_\mu \overline{u}(\vec{p}_1)\Lambda_\mu(q)u(\vec{p}) = 0 \qquad (114)$$

Let us write the first terms of the decomposition for the functions $F_1(q^2)$ and $F_2(q^2)$ over small parameters m^2l^2 and q^2/m^2:

$$F_1(q^2) = \frac{\alpha}{4\pi}\left[\chi - 2\sigma - v'(0) + \frac{9}{2} - 6C - 3m^2l^2v(1)\right] + \frac{\alpha}{2\pi}\frac{q^2}{m^2} \times \left\{\frac{2}{3}\left(\frac{1}{2}\sigma - \frac{3}{8}\right) + \frac{m^2l^2}{3}\left[v(1)\left(-\chi + 2C - \frac{13}{6}\right) + v'(1)\right]\right\} \qquad (115)$$

where $\sigma = \ln(m^2/\mu_{ph}^2)$, $C = 0.577215\ldots$ is the Euler constant, $\alpha = e^2/4\pi$ and $\chi = \ln\left[\frac{1}{m^2l^2}\right]$, and

$$F_2(q^2) = -\frac{\alpha}{2\pi}\left(1 - \frac{2}{3}v(1)m^2l^2\right) \qquad (116)$$

From this last formula we can see that corrections to the anomalous magnetic moment (AMM) due to high-dimensional spacetime with the fundamental length, we call it the Planck constant,for leptons are given by

$$\Delta\mu = \frac{\alpha}{2\pi}\left[1 - \frac{2}{3}v(1)m^2l^2\right] \qquad (117)$$

The first term in (117) corresponds to the Schwinger [14] correction obtained in local QED. The second term is responsible from gravitational effects on the particle physics, where

$$l = L_{Pl} = \sqrt{\frac{G\hbar}{c^3}}.$$

In accordance with form-factors (91), the functions $v(x)$ has the form:

$$v(x) = \frac{1}{2}2^{-2x}\frac{1}{\Gamma(1+x)\Gamma(2+x)}.$$

Finally, notice that if we introduce some other length l which is differ from the Planck length L_{Pl} then for which we can get restriction on its value from the experimental data on measuring of the AMM of leptons. Thus, from experimental values of AMM of the electron and muon [18-20] and [21]:

$$\Delta\mu_{exp}^{(e)} = \frac{\mu_e}{\mu_B} - 1 = \frac{1}{2}(g-2) = \left(1159652180.73(0.28)\right) \times 10^{-12} \qquad (118)$$

and

$$\Delta\mu_{exp}^{(\mu)} = \frac{\mu_\mu}{(e\hbar/2m_\mu)} - 1 = \frac{1}{2}(g_\mu - 2) = \left(116592089(63)\right) \times 10^{-11} \qquad (119)$$

one gets the following restrictions on the value of the universal parameter (or the fundamental length) l:

$$l \leq 7.0 \times 10^{-17} cm \quad \text{for} \quad \Delta\mu_{\text{exp}}^{(e)}, \qquad (120)$$

$$l \leq 2.6 \times 10^{-17} cm \quad \text{for} \quad \Delta\mu_{\text{exp}}^{(\mu)} \qquad (121)$$

Recent very high accuracy experimental result [23] for measuring the anomalous magnetic moment of muon

$$a_\mu(FNAL) = 11659055(24) \times 10^{-11}(0.20ppm) \qquad (122)$$

and

$$a_\mu(exp) = 11659059(22) \times 10^{-11}(0.19ppm) \qquad (123)$$

allows us to compare with the SM calculation [22]

$$a_\mu^{SM}(theory) = 116591810(43) \times 10^{-11}. \qquad (124)$$

As a result difference of these two values is

$$\Delta = a_\mu^{exp} - a_\mu^{SM} = (25.1 \pm 5.9) \times 10^{-10}. \qquad (125)$$

Worth notice that contribution arisen from the high dimensional spacetime to the anomalous magnetic moments of leptons is negative value:

$$(\Delta\mu)_h = -\frac{\alpha}{3\pi}v(1)m^2l^2, \qquad (126)$$

here

$$v(1) = \frac{1}{16}. \qquad (127)$$

D. Vacuum polarization

Since, in this concrete scheme the propagator $S(x-y)$ of the charged lepton spinor does not changed and therefore the diagrams of the vacuum polarization i.e. closed spinor propagators of leptons in the nonlocal QED are investigated by the same way as in the local theory. Therefore, standard calculations by using the D-dimensional regulation procedure (see [28], for detail) read to the following result in $D = 4$ dimensional case:

$$\Pi_{\mu\nu}(q) = \left(q^2 g_{\mu\nu} - q_\nu q_m\right)\Pi(q^2), \qquad (128)$$

here

$$\Pi(q^2) = \frac{e^2}{2\pi^2}\int_0^1 d\,x x(1-x)\ln\left(1 + \frac{q^2 x(1-x)}{m^2}\right) \qquad (129)$$

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