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Thermoelastic Interactions In Functionally Graded Micropolar Thermoelastic Medium Possessing Cubic Symmetry

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***Abstract*— The present investigation deals with two-dimensional problem in a non-homogeneous, micropolar thermoelastic medium possess cubic symmetry as a result of normal and tangential load. The non-homogeneous material properties are supposed to be graded in x-direction. The components of temperature field, displacement components and stresses are obtained by using normal mode technique. The formulation is performed in the context of the Lord-Shulman (LS) as well as Green Lindsay (GL) theories of thermoelasticity. The value of these expressions are calculated numerically and represented graphically for micropolar cubic crystal and micropolar isotropic medium.**

Keywords—Non-Homogeneous, Lord-Shulman, Green-Lindsay, Microrotation, functionally graded, normal mode, cubic symmetry.

1 INTRODUCTION

Study related to generalized thermoelasticity have dragged considerable attention during last few decades due to its application in various practical aspects of life processes such as earthquake prediction, exploration of minerals, soil dynamics etc. First of all, uncoupled classical theory of thermoelasticity which supposed infinite speed of heat propagation was replaced by Biot [1] by considering theory of coupled thermoelasticity. Subsequently, generalized theories of thermoelasticity were developed by Lord and Shulman [2], Green and Lindsay [3], which were further assessed by Green and Nagdhi [4], Hetnarski and Ignaczak [5], and Ignaczak and Ostoja- Starzewski [6]. The linear theory of elasticity has gained much attention in the stress analysis of steel, which is frequently used material in engineering. The other common solid materials like concrete, wood, and coal barely explains the mechanical behavior of linear elasticity. The linear theory of thermoelasticity does not apply on many of new synthetic materials of the clastomer and polymer type. To express the behavior of such materials, linear theory of micropolar of elasticity is sufficient. The linear theory of micropolar elasticity was developed by Eringen and Suhubi [7] and Eringen [8]. Eringen [9] and Nowacki [10] established the linear theory of micropolar thermoelastic in continuation with the theory of micropolar continua which includes thermal effects. Different problems of micropolar theory of thermoelasticity was discussed by various authors [11], [12], [13], [14]. Kumar and Ailawalia [15,16] discussed the behavior and deformation in cubic crystal due to various sources. Kumar and Ailawalia [17] obtained the analytic expressions in micropolar thermoelastic medium which possesses cubic symmetry with one relaxation time. The thermomechanical interactions in the context of two-temperature generalized thermoelasticity are analyzed by Lotfy and Hassan [18] under different types of heating. Kumar and Choudhary [19,20] have discussed various problems in orthotropic micropolar continua. Kumar and Ailawalia [21,22] studied the response of moving inclined load in micropolar theory of elasticity. The study of propagation of plane waves in micropolar thermos-diffusion elastic half-space in the context of generalized theories of thermoelasticity was presented by Kumar, Kaushal, Marin [23]. Deswal, Punia, Kalkal [24] discussed the effects of gravity field and micro polarity on wave propagation in a two-temperature generalized thermoelastic medium within the framework of dual-phase-lag model. Othman, Abo-Dahab, Alosaimi [25] studied the effect of inclined load and magnetic field in a micropolar thermoelastic medium possessing cubic symmetry.

The concept of functionally graded materials (FGMs) was first introduced in 1984, in Japan during a space plane project. These materials are nonhomogeneous in which there is a variation of composition with position resulting in variation of material properties. In FGMs, elastic coefficients are no longer constant but are function of position. These materials are used as a thermal barrier due to their capability to withstand high temperature. Due to their excellent thermo-mechanical properties, they are widely used in aerospace, nuclear reactors, pressure vessels pipes and in chemical plants etc. In many applications, FGMs are found to be better substitute for the conventional homogeneous materials. Hence, the investigation of functionally graded materials have become a very active research area in the field of thermoelasticity. Wang and Mai [26] analyzed the one-dimensional transient temperature and thermal stress fields in nonhomogeneous materials such as plates, cylinders and spheres using a finite element method. Ootao and Tanigawa [27] studied a one-dimensional transient thermoelastic problem of a FGM hollow cylinder whose thermoelastic constants were assumed to vary with the power product form of a radial coordinate variable. Shao, Wang, Ang [28] solved a thermo-mechanical problem of an FGM hollow circular cylinder whose material properties were assumed to be temperature independent and vary continuously in the radial direction. Darabseh, Yilmaz, Bataineh [29] developed the idea of thermoelastic response of thick hollow cylinder made of functionally graded material subjected to thermal load in the context of GL theory. The problem of magneto-thermoelastic interactions in a functionally graded isotropic, unbounded rotating medium with a periodically varying heat source in the context of linear generalized thermoelasticity was discussed by Pal, Das, Kanoria [30]. Using Laplace transform method, Sherief and El-Latief [31] solved the problem of FGM thermoelastic half-space in which Lame's moduli are the functions of vertical distance from the surface of the medium.

The behavior of displacement, stress and temperature field in a ceramic FGM layer under uniform thermal shock was investigated by Nikolarakis and Theotokoglou [32] in the context of LS theory. Purkait, Sur, Kanoria [33] discussed the effects of gravity and magnetic field on a functionally graded thermoelastic half-space under GN theory. Abbas [34] discussed the thermoelastic interactions in an infinite fiber-reinforced anisotropic medium with a circular hole in the context of generalized theory of thermoelasticity. Abbas [35] presented a study on the natural frequencies, thermoelastic damping, and frequency shift of a thermoelastic hollow sphere in the context of the generalized thermoelasticity theory with one relaxation time. The forced vibrations of nonhomogeneous thermoelastic, isotropic, thin annular disk under periodic and exponential types of axisymmetric dynamic pressures were analyzed by Mishra, Sharma, and Sharma [36]. By using the Fourier and Laplace transforms, Xue, Tian, and Liu [37] investigated the effects of time delay, kernel function and non-homogeneity parameter in a functionally graded thermoelastic half-space with memory-dependent heat conduction model. Sur and Kanoria [38] studied a one-dimensional problem of a functionally graded fiber-reinforced thermoelastic medium in the context of TPL model. Sheokand, Kalkal, Deswal [39] investigated the disturbances in a functionally graded thermoelastic medium with DPL model under the effect of gravity and rotation.

The aim of present research is to determine displacement, force stress, couple stress and temperature components in a non-homogeneous, micropolar thermoelastic functionally graded solid with cubic symmetry. The comparisons have been shown for the micropolar cubic crystal and micropolar isotropic solid.

2 *Basic Equations*

Following Minagawa, Arakawa, Yamada [40], Green and Lindsay [3], the constitutive components of stress and couple stress for the micropolar generalized thermoelastic solid with cubic symmetry in the absence of body forces are:

$$\sigma_{xx} = \left[A_1 \frac{\partial u}{\partial x} + A_2 \frac{\partial v}{\partial y} - \nu\left(T + t_1 \frac{\partial T}{\partial t}\right)\right], \tag{1}$$

$$\sigma_{yy} = \left[A_1 \frac{\partial v}{\partial y} + A_2 \frac{\partial u}{\partial x} - \nu\left(T + t_1 \frac{\partial T}{\partial t}\right)\right], \tag{2}$$

$$\sigma_{xy} = \left[A_4\left(\frac{\partial u}{\partial y} - \phi_3\right) + A_3\left(\frac{\partial v}{\partial x} + \phi_3\right)\right], \tag{3}$$

$$m_{ij} = B_1 \phi_{p,p}\delta_{ij} + B_2 \phi_{i,j} + B_3 \phi_{j,i}. \tag{4}$$

Stress equation of motion:

$$\sigma_{ji,j} = \rho \ddot{u}_i\,. \tag{5}$$

Couple stress equation of motion:

$$m_{ip,i} + \epsilon_{ijp}\sigma_{ij} = \rho j \ddot{\phi}_p \tag{6}$$

Equation of heat conduction

$$K^*\nabla^2 T - \rho C^*\left(n_1 + t_0 \frac{\partial}{\partial t}\right)\dot{T} = \nu T_0\left(n_1 + n_0 t_0 \frac{\partial}{\partial t}\right)\dot{e}, \tag{7}$$

Where A_1, A_2, A_3, A_4 are elastic constant, $\nu = (A_1 + 2A_2)\alpha_T$, α_T is the coefficient of linear expansion, σ_{ij} are the components of stress, m_{ij} are the components of couple stress, ρ is the density, T is the absolute temperature, j is the microinertia, K^* is the coefficient of thermal conductivity, $\vec{u}$ is the displacement vector, $\vec{\phi}$ is the microinertia vector , C^* is the specific heat at constant strain; B_1, B_2, B_3 are the micropolar material constants, t_0 and t_1 are the thermal relaxation times and δ_{ij} is the Kronecker delta.

3 *Formulation of the Problem:*

We consider a non-homogeneous, micropolar generalized thermoelastic half space with cubic symmetry. A rectangular coordinate system (x, y, z) having the origin on the surface $x = 0$and the x-axis pointing vertically into the medium is assumed. The present study is restricted to xy plane with displacement vector $\vec{u} = (u, v, 0)$ and the microrotation vector is $\vec{\phi} = (0,0,\phi_3)$ and thus all the field quantities are independent of the space variable z.

For a functionally graded composite, the parameters $A_1,\ A_2,\ A_3,\ A_4, B_3, \nu, \rho, K^*$ are no longer constant but become space-dependent. Hence, we replace $A_1,\ A_2,\ A_3,\ A_4, B_3,\ \nu,\ \rho,\ K^*$ by $A_{10}\overrightarrow{f(x)}, A_{20}\overrightarrow{f(x)}, A_{30}\ \overrightarrow{f(x)}, A_{40}\overrightarrow{f(x)}$, $B_{30}\ \overrightarrow{f(x)}, \nu_0\ \overrightarrow{f(x)}$, $\rho_0\ \overrightarrow{f(x)}$, $K_0^*\overrightarrow{f(x)}$ respectively, $A_{10},\ A_{20},\ A_{30}, A_{40}$, B_{30}, $\nu_0,\ \rho_0,\ K_0^*$ are supposed to be constant and $\overrightarrow{f(x)}$ is a

given non-dimensional function of the space variable $\vec{x} = (x, y, z)$. It is also assumed that material properties vary only in *x*-direction. Hence, we take $f(\vec{x})$ as *f(x)*.
Using these values, the relations (1)-(7) reduces to:

$$\sigma_{xx} = f(x)\left[A_{10}\frac{\partial u}{\partial x} + A_{20}\frac{\partial v}{\partial y} - \nu_0\left(T + t_1\frac{\partial T}{\partial t}\right)\right], \quad (8)$$

$$\sigma_{yy} = f(x)\left[A_{10}\frac{\partial v}{\partial y} + A_{20}\frac{\partial u}{\partial x} - \nu_0\left(T + t_1\frac{\partial T}{\partial t}\right)\right], \quad (9)$$

$$\sigma_{xy} = f(x)\left[A_{40}\left(\frac{\partial u}{\partial y} - \phi_3\right) + A_{30}\left(\frac{\partial v}{\partial x} + \phi_3\right)\right], \quad (10)$$

$$m_{xz} = f(x)\left[B_{30}\frac{\partial \phi_3}{\partial x}\right], \quad (11)$$

$$m_{yz} = f(x)\left[B_{30}\frac{\partial \phi_3}{\partial y}\right], \quad (12)$$

Stress equation of motion:

$$\sigma_{ji,j} = \rho_0 f(x)\ddot{u}_i . \quad (13)$$

Couple stress equation of motion:

$$m_{ip,i} + \epsilon_{ijp}\sigma_{ij} = \rho_0 f(x) j\ddot{\phi}_p, \quad (14)$$

Equation of heat conduction:

$$(K_0^* f(x) T_{,i})_{,i} - f(x)\rho_0 C^*\left(n_1 + t_0\frac{\partial}{\partial t}\right)\dot{T} = f(x)\nu_0 T_0\left(n_1 + t_0 n_0\frac{\partial}{\partial t}\right)\dot{e}. \quad (15)$$

Where, $\nu_0 = (A_{10} + 2A_{20})\alpha_T$
Here, the superposed dot denotes derivatives with respect to time and comma denotes derivative with respect to space variable.
Substituting equations (8)-(11) in (13)-(14), equations of motion are obtained as:

$$f(x)\left[A_{10}\frac{\partial^2 u}{\partial x^2} + (A_{20} + A_{40})\frac{\partial^2 v}{\partial x \partial y} + A_{30}\frac{\partial^2 u}{\partial y^2} + (A_{30} - A_{40})\frac{\partial \phi_3}{\partial y} - \nu_0\left(1 + t_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial x}f(x)\left[A_{10}\frac{\partial u}{\partial x} + A_{20}\frac{\partial v}{\partial y} - \nu_0\left(1 + t_1\frac{\partial}{\partial t}\right)T\right] = \rho_0 f(x)\ddot{u} \quad (16),$$

$$f(x)\left[A_{10}\frac{\partial^2 v}{\partial y^2} + (A_{20} + A_{40})\frac{\partial^2 u}{\partial x \partial y} + A_{30}\frac{\partial^2 v}{\partial x^2} + (A_{30} - A_{40})\frac{\partial \phi_3}{\partial x} - \nu_0\left(1 + t_1\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial x}f(x)\left[A_{40}\frac{\partial u}{\partial y} + (A_{30} - A_{40})\phi_3 + A_{30}\frac{\partial v}{\partial x}\right] = \rho_0 f(x)\ddot{v} \quad (17)$$

$$B_{30}\left[f(x)\nabla^2\phi_3 + \frac{\partial}{\partial x}f(x)\frac{\partial \phi_3}{\partial x}\right] + (A_{30} - A_{40})f(x)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - 2(A_{30} - A_{40})f(x)\phi_3 = \rho_0 f(x) j\ddot{\phi}_3 , \quad (18)$$

The heat conduction equation in non-homogeneous medium reduces to:

$$K_0^*\left[f(x)\nabla^2 T + \frac{\partial}{\partial x}f(x)\frac{\partial T}{\partial x}\right] - f(x)\rho_0 C^*\left(n_1 + t_0\frac{\partial}{\partial t}\right)\dot{T} = f(x)\nu_0 T_0\left(n_1 + t_0 n_0\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right). \quad (19)$$

Where,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

4 *Exponential Variation:*

We consider $f(x) = e^{-nx}$, where *n* is non dimensional parameter. Hence, the material properties vary exponentially along the *x*-direction. To rewrite the governing equations in a dimensionless form, introduce the following non-dimensional parameters:

$$\overline{x_i} = \frac{\omega^*}{C_0}x_i,\quad \overline{u_i} = \frac{\rho_0 C_0 \omega^*}{\nu_0 T_0}u_i,\quad \overline{t_0} = \omega^* t_0,\quad \overline{t} = \omega^* t,\quad \overline{T} = \frac{T}{T_0},$$
$$\overline{t_1} = \omega^* t_1,\quad \overline{\phi_3} = \frac{\rho_0 C_0^2}{\nu_0 T_0}\phi_3,\quad \overline{\sigma_{ij}} = \frac{\sigma_{ij}}{\nu_0 T_0},\quad \overline{m_{ij}} = \frac{\omega^*}{\nu_0 C_0 T_0}m_{ij},$$
$$\omega^* = \frac{\rho_0 C_0^2 C^*}{K^*},\quad C_0^2 = \frac{A_{10}}{\rho_0}. \quad (20)$$

Equations (8)-(12) and (16)-(19) take the following non-dimensional form (after dropping dashes for convenience)

$$\sigma_{xx} = e^{-nx}\left[\frac{\partial u}{\partial x} + l_4\frac{\partial v}{\partial y} - \left(1 + t_1\frac{\partial}{\partial t}\right)T\right], \quad (21)$$

$$\sigma_{yy} = e^{-nx}\left[l_4\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - \left(1 + t_1\frac{\partial}{\partial t}\right)T\right], \quad (22)$$

$$\sigma_{xy} = e^{-nx}\left(l_5\frac{\partial u}{\partial y} + l_2\frac{\partial v}{\partial x} + l_3\phi_3\right), \quad (23)$$

$$m_{xz} = \left[l_6\, e^{-nx}\frac{\partial \phi_3}{\partial x}\right], \quad (24)$$

$$m_{yz} = \left[l_6 e^{-nx}\frac{\partial \phi_3}{\partial y}\right], \quad (25)$$

$[\frac{\partial^2 u}{\partial x^2} + l_1 \frac{\partial^2 v}{\partial x \partial y} + l_2 \frac{\partial^2 u}{\partial y^2} + l_3 \frac{\partial \phi_3}{\partial y} - \left(1 + t_1 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x}] - n\, [\frac{\partial u}{\partial x} + l_4 \frac{\partial v}{\partial y} - \left(1 + t_1 \frac{\partial}{\partial t}\right) T] = \frac{\partial^2 u}{\partial t^2}$, (26)

$[\frac{\partial^2 u_2}{\partial y^2} + l_1 \frac{\partial^2 u}{\partial x \partial y} + l_2 \frac{\partial^2 v}{\partial x^2} + l_3 \frac{\partial \phi_3}{\partial x} - \left(1 + t_1 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial y}] - n\, [l_5 \frac{\partial u}{\partial y} + l_2 \frac{\partial v}{\partial x} + \phi_3 l_3] = \frac{\partial^2 v}{\partial t^2}$, (27)

$l_6\, [\nabla^2 \phi_3 - n \frac{\partial \phi_3}{\partial x}] + l_3 (\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}) - 2 l_3 \phi_3 = l_7 \frac{\partial^2 \phi_3}{\partial t^2}$, (28)

$[\nabla^2 T - n \frac{\partial T}{\partial x}] - l_8 \left(n_1 + t_0 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} = l_9 \left(n_1 + t_0 n_0 \frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0$ (29)

where, $l_1 = \frac{A_{20}+A_{40}}{\rho_0 C_0^2}$, $l_2 = \frac{A_{30}}{\rho_0 C_0^2}$, $l_3 = \frac{A_{30}-A_{40}}{\rho_0 C_0^2}$, $l_4 = \frac{A_{30}}{\rho_0 C_0^2}$, $l_5 = \frac{A_{40}}{\rho_0 C_0^2}$, $l_6 = \frac{B_{30}\omega^{*2}}{\rho_0 C_0^4}$, $l_7 = \frac{j\omega^{*2}}{C_0^2}$, $l_8 = \frac{C_0^2 \rho_0 C^*}{K_0^* \omega^*}$, $l_9 = \frac{v_0^2 T_0}{K_0^* \omega^* \rho_0}$. (30)

5 *Solution of the Problem:*

In this section, normal mode technique is used to obtain the analytical expressions for displacement components, microrotation, stress components, temperature and couple stress. The solution of the considered physical variable can be decomposed in $(u^*, v^*, T^*, \phi_3^*, \sigma_{ij}^*, m_{ij}^*)$ terms of normal mode in the following form:

$$(u, v, T, \phi_3, \sigma_{ij}, m_{ij}) = (u^*, v^*, T^*, \phi_3^*, \sigma_{ij}^*, m_{ij}^*)(x) e^{\omega t + \iota b y} \qquad (31)$$

where $u^*, v^*, T^*, \phi_3^*, \sigma_{ij}^*, m_{ij}^*$ are the amplitudes of the functions, ω is the angular frequency, ι is the imaginary unit, and *b* is the wave number in *y* direction. Using equation (31) in equations (26)-(29), we obtain

$$(D^2 - nD - a_1)u^* + (a_2 D - a_3)v^* - a_4(D - n)T^* + a_5 \phi_3^* = 0\,, \qquad (32)$$

$$(a_2 D - a_6)u^* + (l_2 D^2 - n l_2 D - a_7)v^* - a_8 T^* + (l_3 D - a_9)\phi_3^* = 0, \qquad (33)$$

$$a_{10} u^* - l_3 D v^* + (D^2 - nD - a_{11})\phi_3^* = 0\,, \qquad (34)$$

$$a_{12} D u^* + a_{13} v^* - (D^2 - nD - a_{14})T^* = 0. \qquad (35)$$

Where,
$D = \frac{d}{dx}$, $a_1 = l_2 b^2 + \omega^2$, $a_2 = \iota b l_1$, $a_3 = \iota b n l_4$, $a_4 = (1 + t_1\omega)$, $a_5 = l_3 \iota b$, $a_6 = \iota b n l_5$, $a_7 = b^2 + \omega^2$, $a_8 = (1 + t_1\omega)\iota b$, $a_9 = n l_3$, $a_{10} = \iota b l_3$, $a_{11} = l_6 b^2 + 2 l_3 + l_7 \omega^2$, $a_{12} = l_9(n_1\omega + n_0 t_0 \omega^2)$, $a_{13} = l_9 \iota b (n_1 \omega + n_0 t_0 \omega^2)$, $a_{14} = b^2 + l_8(n_1\omega + t_0\omega^2)$.

The condition for the existence of a non-trivial solution of the system of equations (32)-(35) provides us

$$(D^8 + N_1 D^7 + N_2 D^6 + N_3 D^5 + N_4 D^4 + N_5 D^3 + N_6 D^2 + N_7 D + N_8)(u^*, T^*, \phi_3^*, T^*) = 0 \qquad (36)$$

Where $N_i (i = 1,2, \ldots ,8)$ are listed in Appendix *A*.

The solution of equation (36) which is bounded as $x \to \infty$, is given by

$$u^*(x) = \sum_{i=1}^{4} M_i(a,\omega) e^{-k_i x}, \qquad (37)$$

$$T^*(x) = \sum_{i=1}^{4} J_{1i} M_i(a,\omega) e^{-k_i x}, \qquad (38)$$

$$\phi_3{}^*(x) = \sum_{i=1}^{4} J_{2i} M_i(a,\omega) e^{-k_i x}, \qquad (39)$$

$$v^*(x) = \sum_{i=1}^{4} J_{3i} M_i(a,\omega) e^{-k_i x}, \qquad (40)$$

Where $k_i's$ $(i = 1,2,3,4)$ are the roots of the equation (36) and $M_i(a,\omega)$ $(i = 1,2,3,4)$ are the parameters, depending upon a and ω, and

$$J_{1i} = \frac{t_{32} k_i^3 - t_{33} k_i^2 - t_{34} k_i - t_{35}}{t_{36} k_i^4 + t_{37} k_i^3 + t_{38} k_i^2 - t_{39} k_i + t_{40}}, \qquad (41)$$

$$J_{2i} = \frac{(a_2 k_i^3 + t_8 k_i^2 + t_9 k_i - t_{10}) J_{1i} - (t_5 k_i^2 - t_6 k_i - t_7)}{t_{11}}, \qquad (42)$$

$$J_{3i} = \frac{-[a_{10} + (k_i^2 + n k_i - a_{11}) J_{2i}]}{l_6 k_i}, \quad i = (1,2,3,4) \qquad (43)$$

In view of solutions given by (37)-(40), the stress and couple stress components are obtained as

$$\sigma_{xx}^* = e^{-nx} \sum_{i=1}^{4} M_i(a,\omega) e^{-k_i x} W_i, \qquad (44)$$

$$\sigma_{yy}^* = e^{-nx} \sum_{i=1}^{4} M_i(a,\omega) e^{-k_i x} U_i, \qquad (45)$$

$$\sigma_{xy}^* = e^{-nx} \sum_{i=1}^{4} M_i(a,\omega) e^{-k_i x} V_i, \qquad (46)$$

$$m_{xz}^* = e^{-nx} l_6 \sum_{i=1}^{4} J_{2i} M_i(a,\omega) e^{-k_i x}, \qquad (47)$$

Where,

$$W_i = -k_i + l_4 \iota b J_{3i} - a_4 J_{1i} \qquad (48)$$

$$U_i = -l_4 k_i + \iota b J_{3i} - a_4 J_{1i} \qquad (49)$$

$$V_i = -k_i l_5 + l_2 \iota b J_{3i} + l_3 J_{2i} \qquad (50)$$

6. Application:

To determine the constants $M_i(a,\omega)$ $(i=1,2,3,4)$, the boundary conditions at the free surface $x=0$ are given by:

$$a)\ \sigma_{xx} = -F_1 e^{\omega t+\iota by}, \quad (51)$$
$$b)\ \sigma_{xy} = -F_2 e^{\omega t+\iota by}, \quad (52)$$
$$c)\ m_{xz} = 0, \quad (53)$$
$$d)\ \frac{\partial T}{\partial x} = 0, \quad (54)$$

Where F_1 is a normal line load acting in the positive *x*-direction and F_2 is the tangential load acting at the origin in the positive *y* direction.

Using equations (37)-(40) and (44)-(47) in the boundary conditions (51)-(54), we get four equations in four unknowns as:

$$W_1M_1 + W_2M_2 + W_3M_3 + W_4M_4 = -F_1, \quad (55)$$
$$V_1M_1 + V_2M_2 + V_3M_3 + V_4M_4 = -F_2, \quad (56)$$
$$P_1M_1 + P_2M_2 + P_3M_3 + P_4M_4 = 0, \quad (57)$$
$$Q_1M_1 + Q_2M_2 + Q_3M_3 + Q_4M_4 = 0, \quad (58)$$

Where,

$$P_i = l_6 J_{2i}, \quad Q_i = k_i J_{1i}$$

Equations (55)-(58), may be expressed in the matrix form as:

$$\begin{matrix} W_1 & W_2 & W_3 & W_4 \\ V_1 & V_2 & V_3 & V_4 \\ P_1 & P_2 & P_3 & P_4 \\ Q_1 & Q_2 & Q_3 & Q_4 \end{matrix} \quad \begin{matrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{matrix} = \begin{matrix} -F_1 \\ -F_2 \\ 0 \\ 0 \end{matrix} \quad (59)$$

Solution of the system (59) provides us the values of $M_i (i=1,2,3,4)$ as follows:

$$M_i = \frac{\Delta_i}{\Delta}, \quad (i=1,2,3,4) \quad (60)$$

Where Δ and Δ_i are given in Appendix *B*.

Substituting (60) into expressions (37)-(40) and (44)-(47), we get the expressions for displacement components, temperature distribution, microrotation, force stress and couple stress for a functionally graded micropolar thermoelastic medium with cubic symmetry as:

$$(u^*, T^*, \phi_3^*, v^*)(x) = \frac{1}{\Delta}\sum_{i=1}^{4}(1, J_{1i}, J_{2i}, J_{3i})\Delta_i e^{-k_i x}, \quad (61)$$

$$(\sigma_{xx}^*, \sigma_{yy}^*, \sigma_{xy}^*, m_{xz}^*)(x) = \frac{1}{\Delta}\sum_{i=1}^{4}(W_i, U_i, V_i, P_i)\Delta_i e^{-k_i x - nx}. \quad (62)$$

7. Particular Case:

Substituting, $A_1 = \lambda + 2\mu + k$, $A_2 = \lambda$, $A_3 = \mu + k$, $A_4 = \mu$, $B_3 = \gamma$ we obtain the results for thermoelastic micropolar isotropic medium. Where λ, μ, k, γ are elastic constants of the medium.

8. Numerical Results:

For numerical computations, we take the following values of relevant parameters for a micropolar cubic crystal as:

$A_1 = 19.3 \times 10^{11}\, dyne/cm^2$, $A_2 = 10.2 \times 10^{11}\, dyne/cm^2$, $A_3 = 5.8 \times 10^{11}\, dyne/cm^2$, $A_4 = 4.7 \times 10^{11}\, dyne/cm^2$, $B_3 = 1.1 \times 10^{-4} dyne$.

For the comparison with a micropolar isotropic solid, following Eringen [41] we take the following values of relevant parameters for the case of a Magnesium crystal like material

$\rho = 1.74\ gm/cm^3$, $\lambda = 9.4 \times 10^{11}\, dyne/cm^2$, $\mu = 4 \times 10^{11}\, dyne/cm^2$, $k = 1.0 \times 10^{11}\, dyne/cm^2$, $\gamma = 0.779 \times 10^{-4} dyne$, $j = 0.2 \times 10^{-15} cm^2$, $T_0 = 23℃$, $C^* = 0.23\, cal/gm℃$, $K^* = 0.6 \times 10^{-2}\, cal/cmsec℃$.

9. Discussion:

The numerical values of displacement components *u*, *v* and normal stress components σ_{xx}, σ_{xy} and temperature *T* and the couple stress m_{xz} under normal load for micropolar cubic crystal are shown in Fig. 1 - Fig. 6. These figures represent the solution which is obtained by using the generalized theory with one relaxation time (Lord-Shulman (L-S) theory: $n_0 = 0$, $n_1 = 1$, $t_0 = 0.02$, $t_1 = 0$ and generalized theory with two relaxation times (Green-Lindsay(G-L) theory: $n_0 = 0$, $n_1 = 1$, $t_0 = 0.02$, $t_1 = 0.03$.

In Fig. 1 the tangential displacement under L-S theory for non-homogeneous medium remains stagnant at $0 \le x \le 10$ but increasing at $10 \le x \le 20$ while under G-L theory displacement shows linear trend at beginning but increasing at $12 \le x \le 20$ and at homogeneous medium the variations of displacement under L-S and G-L theory are similar in nature for micro-polar cubic crystal (MCC) medium. From Fig. 2 it is observed that the normal displacement under G-L and L-S theory for homogeneous and non-homogeneous medium under MCC start decreasing after $x = 4$ and follow the same trend till end. In context of two theories Fig. 3 and Fig. 4 indicates steep decrease in the normal stress σ_{xx} and tangential stressσ_{xy} till $x = 1.8$ after which it follows the linear trend at all values of x. In Fig.5, temperature for homogeneous medium is increasing slightly as x increases but in non-homogeneous medium temperature (T) decreases as x increases. With reference to L-S and G-L at $n = 0$, Fig. 6 follows linear trend throughout while for L-S and G-L at $n = 1$ the couple stress m_{xz} shows steep increase in values of x from $x = 0$ to $x = 1.7$ and linear trend afterwards.

Fig.7 - Fig. 12 show the comparison between displacement components u, v, temperature T, stress components σ_{xx}, σ_{xy} and the couple stress m_{xz} under normal load for micropolar isotropic medium. From Fig. 7, it is observed that displacement component u starts increasing at $x = 8$. In Fig. 8, the displacement component v decreases sharply after $x = 6$ for non-homogeneous medium till end. The variations of normal stress σ_{xx} and tangential stress σ_{xy} as shown in Fig. 9 and Fig. 10 are opposite in nature. In Fig. 11, temperature shows linear trend for L-S and G-L theory for homogeneous medium and decreases for L-S and G-L for non-homogeneous medium. In Fig. 12 couple stress m_{xz} follows the same trend as followed by the couple stress for micropolar cubic crystal.

9. Conclusion:

1. An analytic solution of the problem on thermo-elastic micropolar solid with cubic symmetry is developed.
2. The homogeneous and non-homogeneous parameters has a significant effect on all the physical variables under different theories.
3. The variations of normal force stress and tangential force stress are opposite in nature in the beginning.
4. The couple stress of micropolar cubic crystal (MCC) and micropolar isotropic solid (MIS) depicted as the mirror image of each other.

Appendix A

$$\mathrm{N}_1 = \frac{(t_{22}t_{36} - t_{21}t_{37} + t_{33}t_{26} - t_{32}t_{27})}{t_{21}t_{36} + t_{32}t_{26}},$$
$$\mathrm{N}_2 = \frac{(t_{23}t_{36} - t_{22}t_{37} + t_{21}t_{38} - t_{34}t_{26} - t_{33}t_{27} + t_{32}t_{28})}{t_{21}t_{36} + t_{32}t_{26}},$$
$$\mathrm{N}_3 = \frac{(t_{24}t_{36} - t_{23}t_{37} + t_{22}t_{38} + t_{21}t_{39} + t_{35}t_{26} + t_{34}t_{27} + t_{33}t_{28} + t_{32}t_{29}}{t_{21}t_{36} + t_{32}t_{26}},$$
$$N_4 = \frac{(t_{25}t_{36} - t_{24}t_{37} + t_{23}t_{38} + t_{22}t_{39} + t_{21}t_{40} - t_{27}t_{35} - t_{34}t_{28} + t_{33}t_{29} - t_{32}t_{30})}{t_{21}t_{36} + t_{32}t_{26}},$$
$$\mathrm{N}_5 = \frac{(-t_{25}t_{37} + t_{24}t_{38} + t_{23}t_{39} + t_{22}t_{40} + t_{35}t_{28} - t_{134}t_{29} - t_{33}t_{30} - t_{32}t_{31})}{t_{21}t_{36} + t_{32}t_{26}},$$
$$\mathrm{N}_6 = \frac{(t_{25}t_{38} \mp t_{24}t_{39} + t_{23}t_{40} + t_{35}t_{29} + t_{34}t_{30} - t_{33}t_{31})}{t_{21}t_{36} + t_{32}t_{26}},$$
$$\mathrm{N}_7 = \frac{(t_{25}t_{39} + t_{24}t_{40} - t_{35}t_{30} + t_{34}t_{31})}{t_{21}t_{36} + t_{32}t_{26}},$$
$$\mathrm{N}_8 = \frac{(t_{25}t_{40} - t_{35}t_{31})}{t_{21}t_{36} + t_{32}t_{26}},$$

And
$t_1 = a_{12}l_6$, $t_2 = a_{10}a_{13}$, $t_3 = a_{14}l_6$, $t_4 = a_{11}a_{13}$, $t_5 = a_{13} - a_2a_{12}$, $t_6 = a_3a_{12} - na_{13}$, $t_7 = a_1a_{13}$, $t_8 = a_3 + na_2$, $t_9 = na_3 - a_2a_{14} - a_4a_{13}$, $t_{10} = a_3a_{14} + na_4a_{13}$, $t_{11} = a_5a_{13}$, $t_{12} = -a_{12}l_2$, $t_{13} = nl_2a_{12}$, $t_{14} = a_2a_{13} + a_7a_{12}$, $t_{15} = a_6a_{13}$, $t_{16} = (n^2 - a_{14})l_2 - a_7$, $t_{17} = n(l_2a_{14} + a_7)$, $t_{18} = a_8a_{13} + a_7a_{14}$, $t_{19} = l_3a_{13}$, $t_{20} = a_9a_{13}$, $t_{21} = -t_5a_{13}$, $t_{22} = (nt_5 - t_6)a_{13}$, $t_{23} = t_1t_{11} + a_{13}(t_7 + nt_6) + t_5t_{14}$, $t_{24} = t_6t_{14} - nt_7a_{13}$, $t_{25} = t_2t_{11} - t_7t_{14}$, $t_{26} = a_2a_{13}$, $t_{27} = (t_8 + na_2)a_{13}$, $t_{28} = l_6t_{11} + (t_9 + nt_8)a_{13} - a_2t_{14}$, $t_{29} = t_8t_{14} - nl_6t_{11} + (t_{10} - nt_9)a_{13}$, $t_{30} = t_3t_{11} + na_{13}t_{10} + t_9t_{14}$, $t_{31} = t_{10}t_{14}$, $t_{32} = t_5t_{19} - t_{11}t_{12}$, $t_{33} = t_6t_{19} - t_5t_{20} - t_{11}t_{13}$, $t_{34} = t_7t_{19} + t_6t_{20} + t_{11}t_{14}$, $t_{35} = t_7t_{20} + t_{11}t_{15}$, $t_{36} = a_2t_{19} + l_2t_{11}$, $t_{37} = t_8t_{19} + a_2t_{20} + 2nl_2t_{11}$, $t_{38} = t_9t_{19} + t_8t_{20} + t_{11}t_{16}$, $t_{39} = t_{10}t_{19} - t_9t_{20} + t_{11}t_{17}$, $t_{40} = t_{11}t_{18} - t_{10}t_{20}$,

Appendix B

$$\begin{aligned}\Delta = {} & W_1[V_2(P_3Q_4 - Q_3P_4) - V_3(P_2Q_4 - Q_2P_4) \\ & + V_4(P_2Q_3 - Q_2P_3)] \\ & - W_2[V_1(P_3Q_4 - Q_3P_4) \\ & - V_3(P_1Q_4 - Q_1P_4) + V_4(P_1Q_3 - Q_1P_3)] \\ & + W_3[V_1(P_2Q_4 - Q_2P_4) \\ & - V_2(P_1Q_4 - Q_1P_4) + V_4(P_1Q_2 - Q_1P_2)] \\ & - W_4[V_1(P_2Q_3 - Q_2P_3) \\ & - V_2(P_1Q_3 - Q_1P_3) \\ & + V_3(P_1Q_2 - Q_1P_2)],\end{aligned} \tag{63}$$

$$\begin{aligned}\Delta_1 = {} & -F_1[V_2(P_3Q_4 - Q_3P_4) - V_3(P_2Q_4 - Q_2P_4) \\ & + V_4(P_2Q_3 - Q_2P_3) \\ & + F_2[W_2(P_3Q_4 - Q_3P_4) \\ & - W_3(P_2Q_4 - Q_2P_4) \\ & + W_4(P_2Q_3 - Q_2P_3)],\end{aligned} \tag{64}$$

$$\begin{aligned}\Delta_2 = {} & -F_1[V_1(P_3Q_4 - Q_3P_4) - V_3(P_1Q_4 - Q_1P_4) \\ & + V_4(P_1Q_3 - Q_1P_3)] \\ & + F_2[V_1(P_3Q_4 - Q_3P_4) \\ & - V_3(P_1Q_4 - Q_1P_4) \\ & + V_4(P_1Q_3 - Q_1P_3)],\end{aligned} \tag{65}$$

$$\begin{aligned}\Delta_3 = {} & -F_1[V_1(P_2Q_4 - Q_2P_4) - V_2(P_1Q_4 - Q_1P_4) \\ & + V_4(P_1Q_2 - Q_1P_2)] \\ & + F_2[V_1(P_2Q_4 - Q_2P_4) \\ & - V_2(P_1Q_4 - Q_1P_4) \\ & + V_4(P_1Q_2 - Q_1P_2)],\end{aligned} \tag{66}$$

$$\begin{aligned}\Delta_4 = {} & -F_1[V_1(P_2Q_3 - Q_2P_3) - V_2(P_1Q_3 - Q_1P_3) \\ & + V_3(P_1Q_2 - Q_1P_2)] \\ & + F_2[V_1(P_2Q_3 - Q_2P_3) \\ & - V_2(P_1Q_3 - Q_1P_3) \\ & + V_3(P_1Q_2 - Q_1P_2)].\end{aligned} \tag{67}$$

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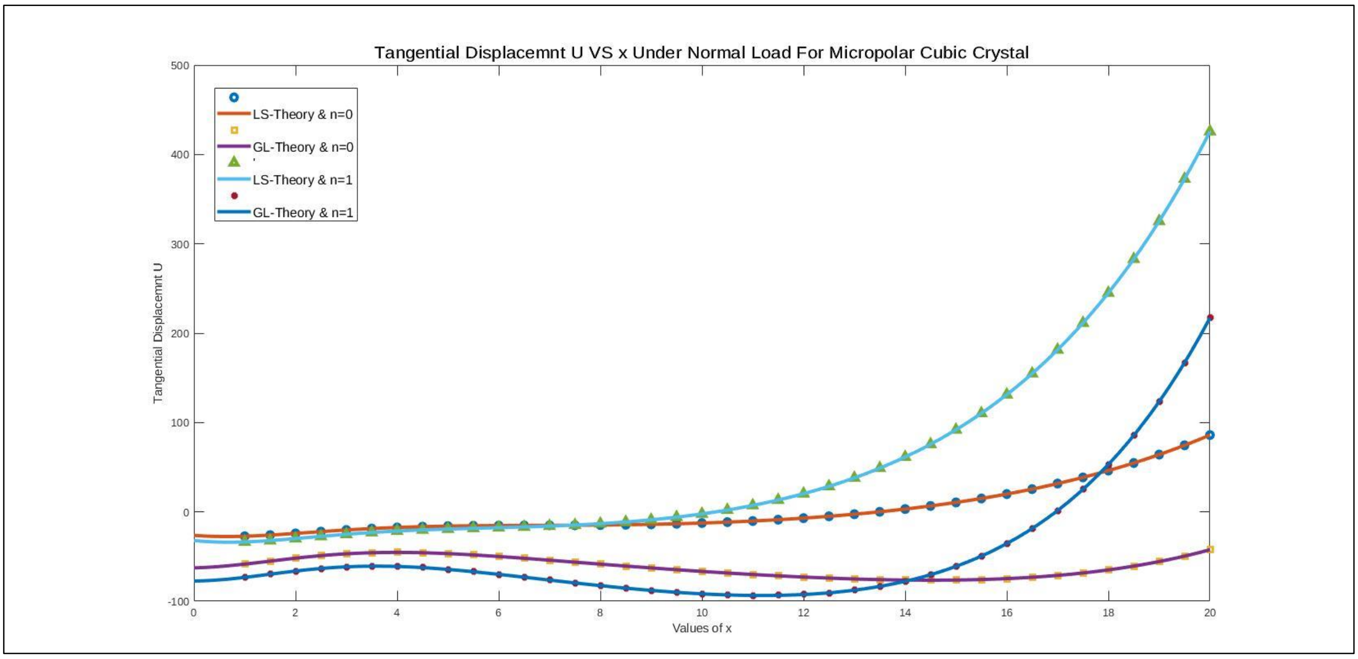
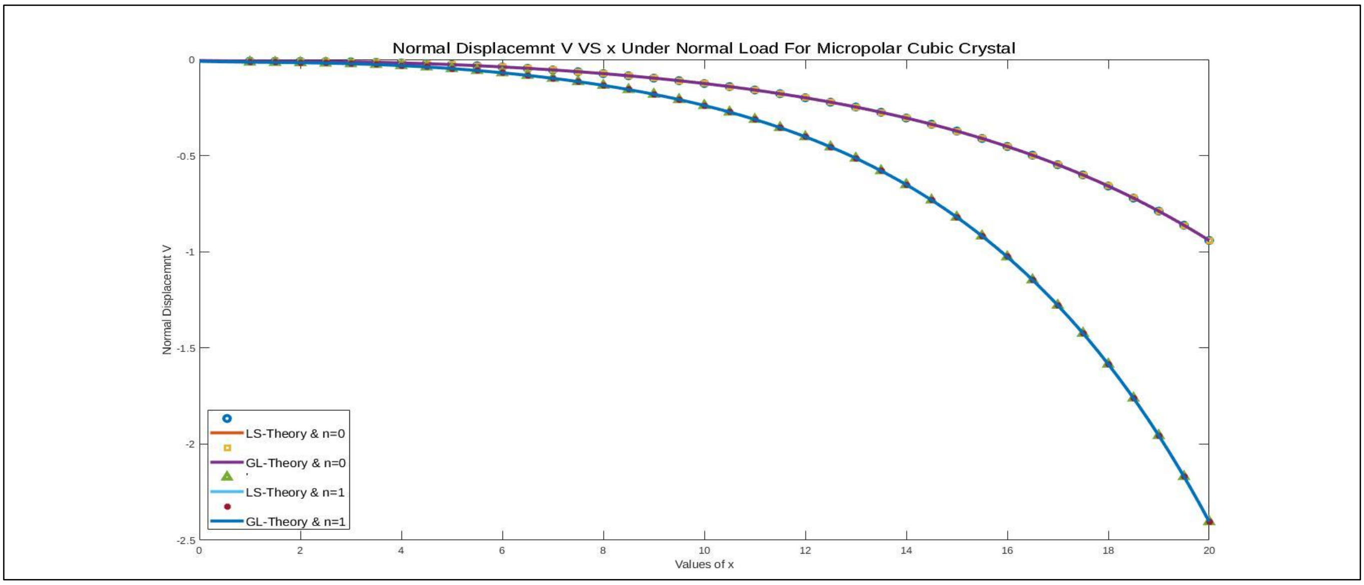
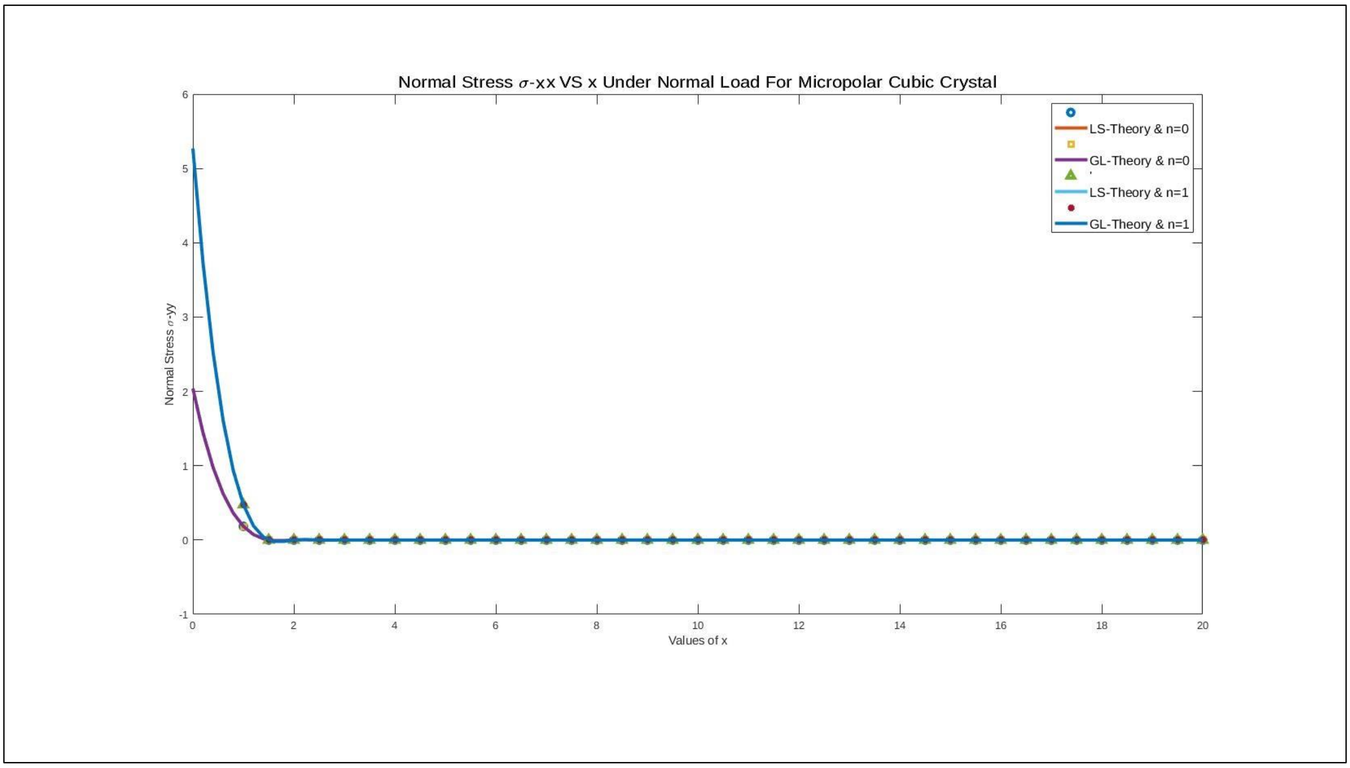
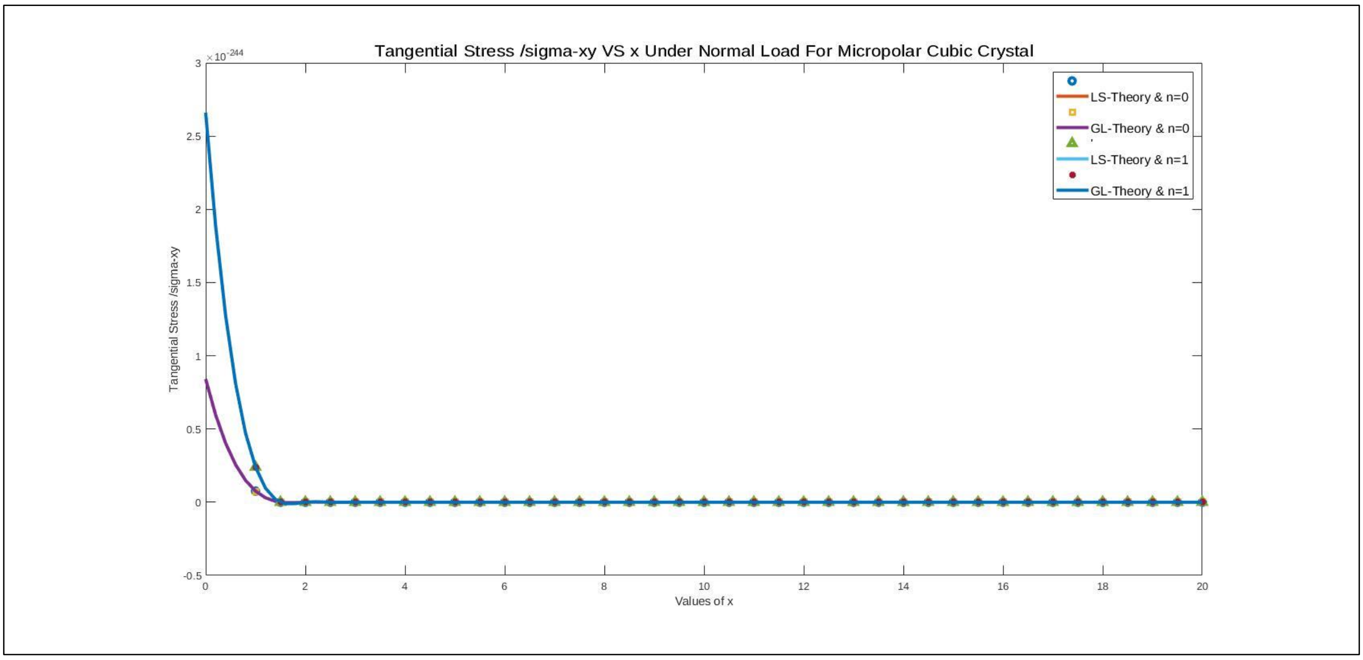
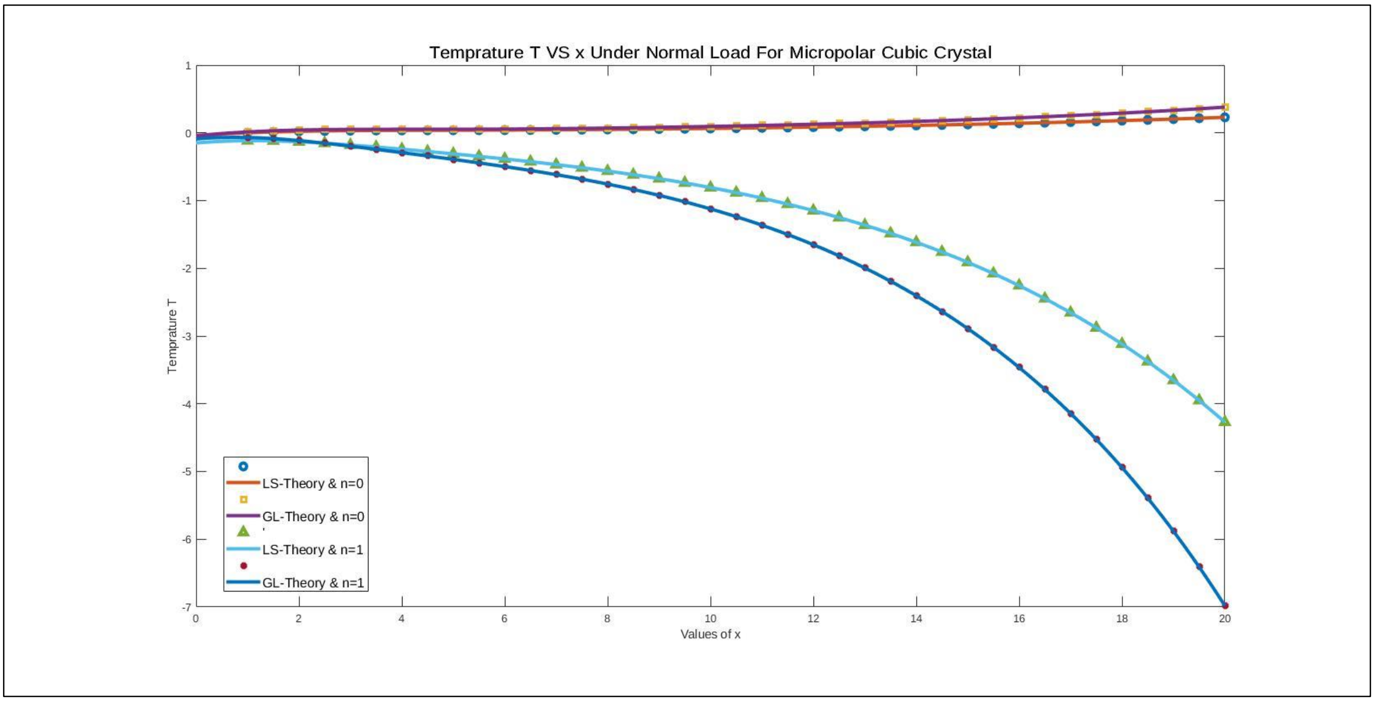
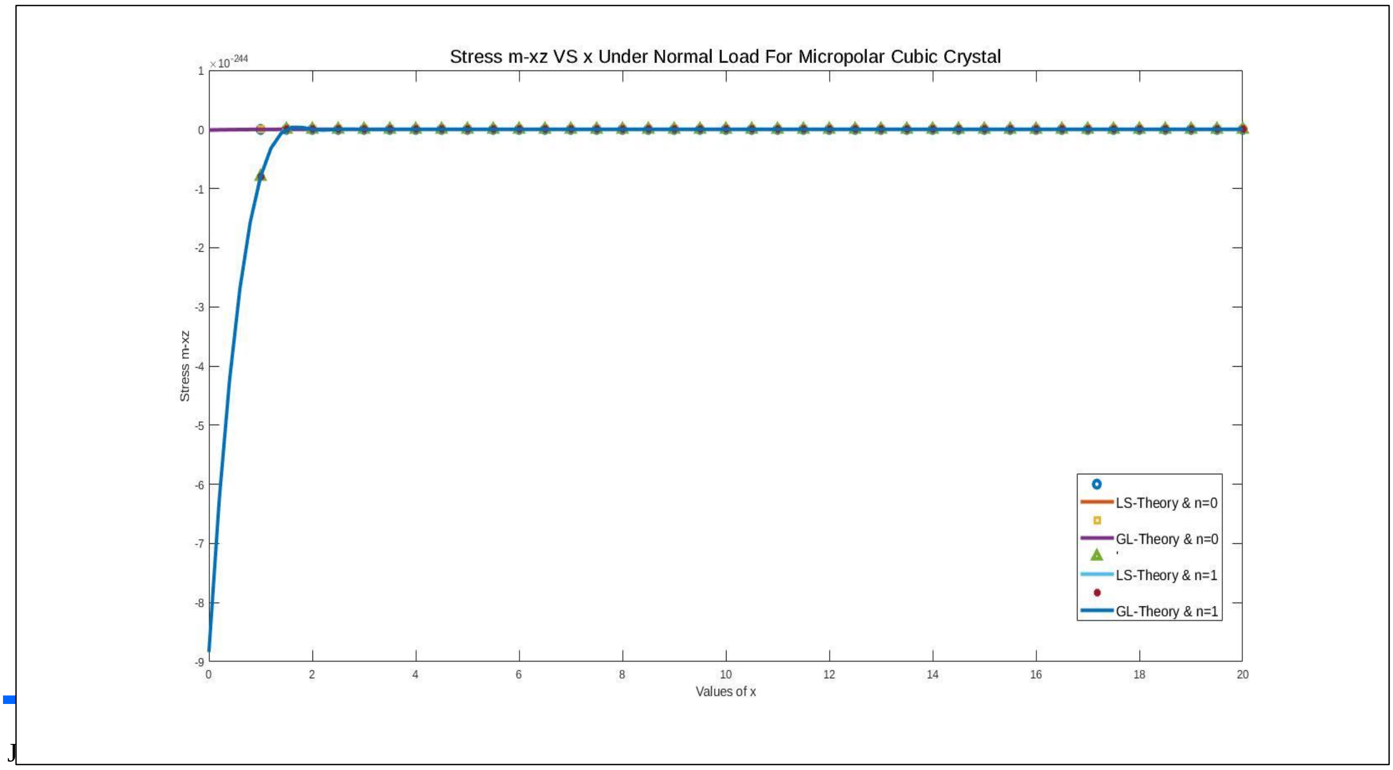





Normal Stress σ-xx VS x Under Normal Load For Micropolar Cubic Crystal
Normal Stress σ-yy
Values of x
LS-Theory & n=0
GL-Theory & n=0
LS-Theory & n=1
GL-Theory & n=1


Tangential Stress /sigma-xy VS x Under Normal Load For Micropolar Cubic Crystal
Tangential Stress /sigma-xy
Values of x
LS-Theory & n=0
GL-Theory & n=0
LS-Theory & n=1
GL-Theory & n=1


Temprature T VS x Under Normal Load For Micropolar Cubic Crystal
Temprature T
Values of x
LS-Theory & n=0
GL-Theory & n=0
LS-Theory & n=1
GL-Theory & n=1


Stress m-xz VS x Under Normal Load For Micropolar Cubic Crystal
$\times 10^{-244}$
Stress m-xz
Values of x
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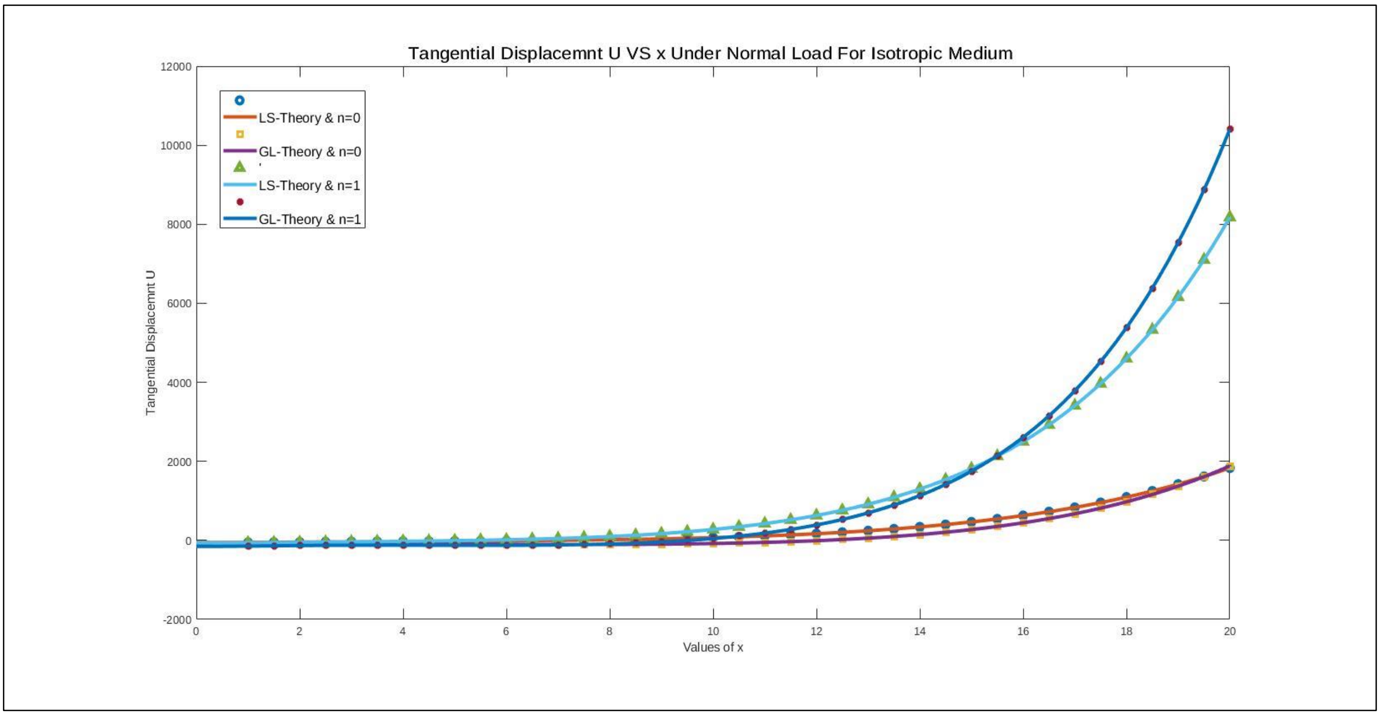
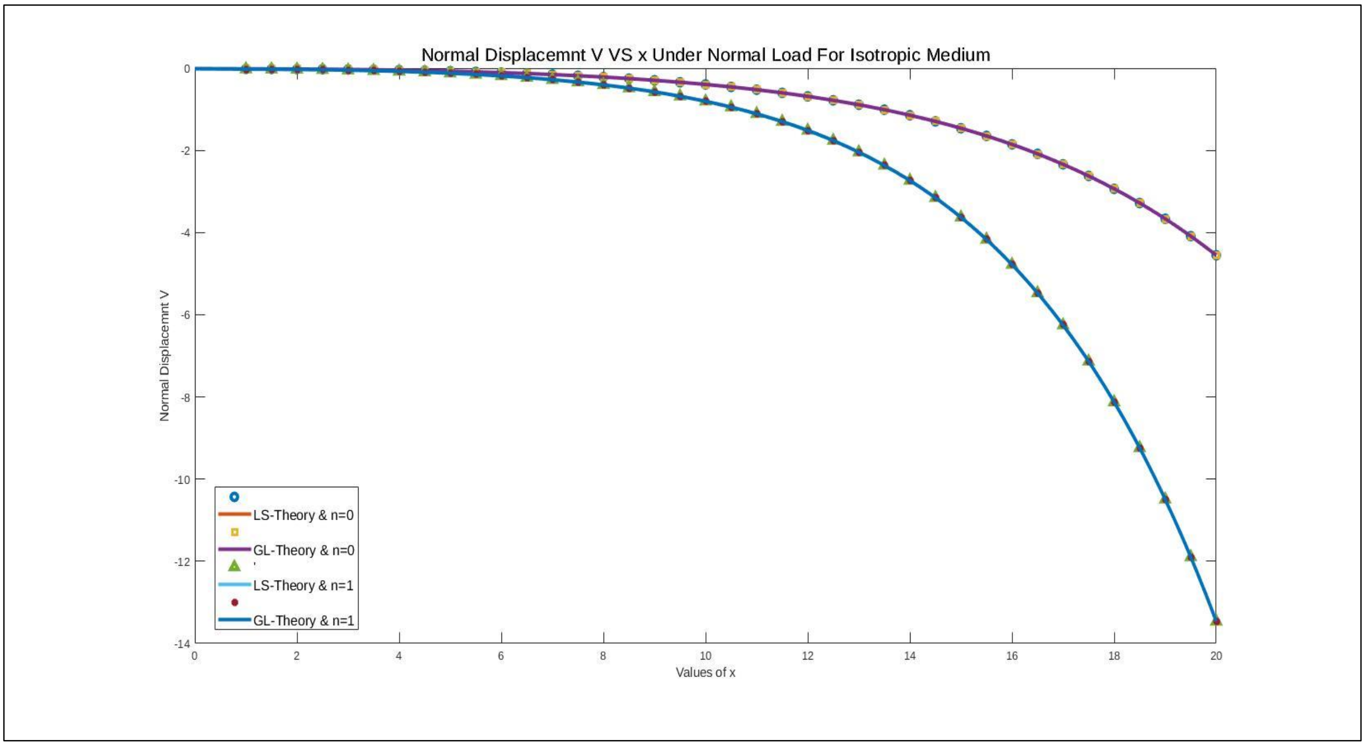
GL-Theory & n=0
LS-Theory & n=1
GL-Theory & n=1


Tangential Displacemnt U VS x Under Normal Load For Isotropic Medium
LS-Theory & n=0
GL-Theory & n=0
LS-Theory & n=1
GL-Theory & n=1
Tangential Displacemnt U
Values of x
12000
10000
8000
6000
4000
2000
0
-2000
0
2
4
6
8
10
12
14
16
18
20


Normal Displacemnt V VS x Under Normal Load For Isotropic Medium
LS-Theory & n=0
GL-Theory & n=0
LS-Theory & n=1
GL-Theory & n=1
Normal Displacemnt V
Values of x
0
-2
-4
-6
-8
-10
-12
-14
0
2
4
6
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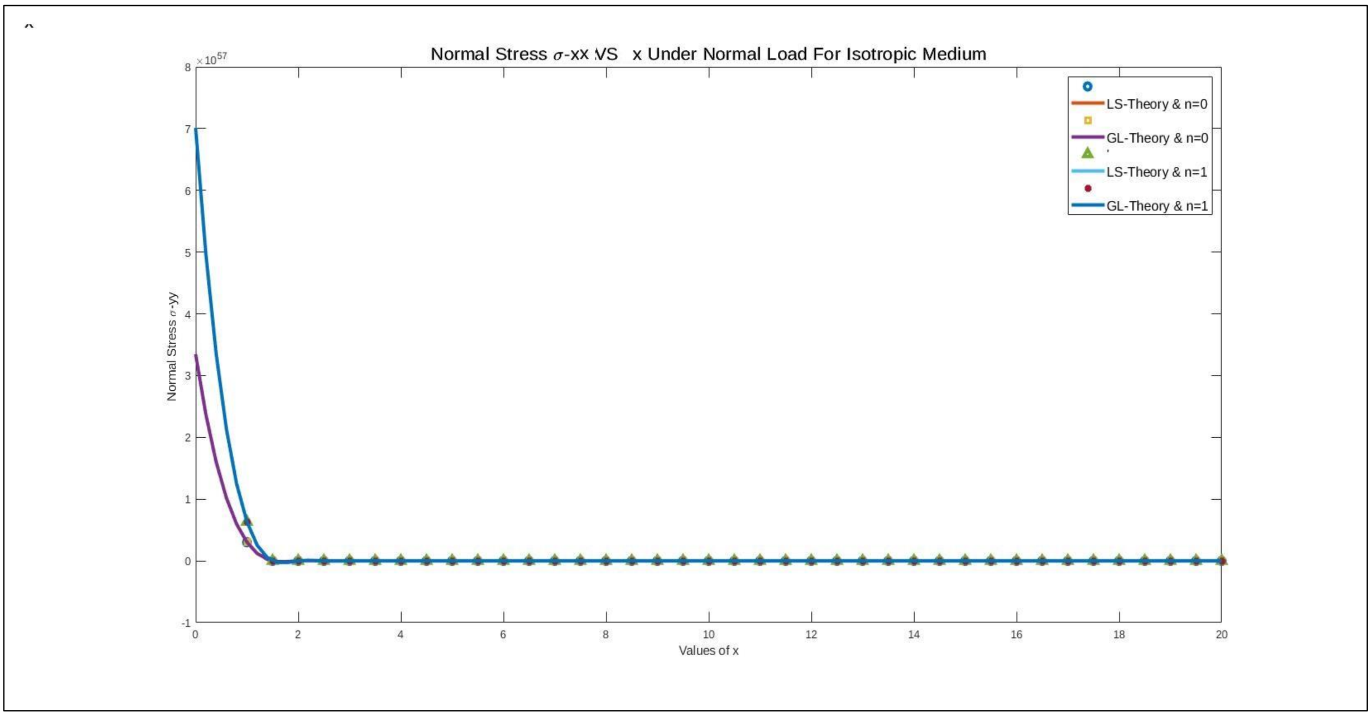
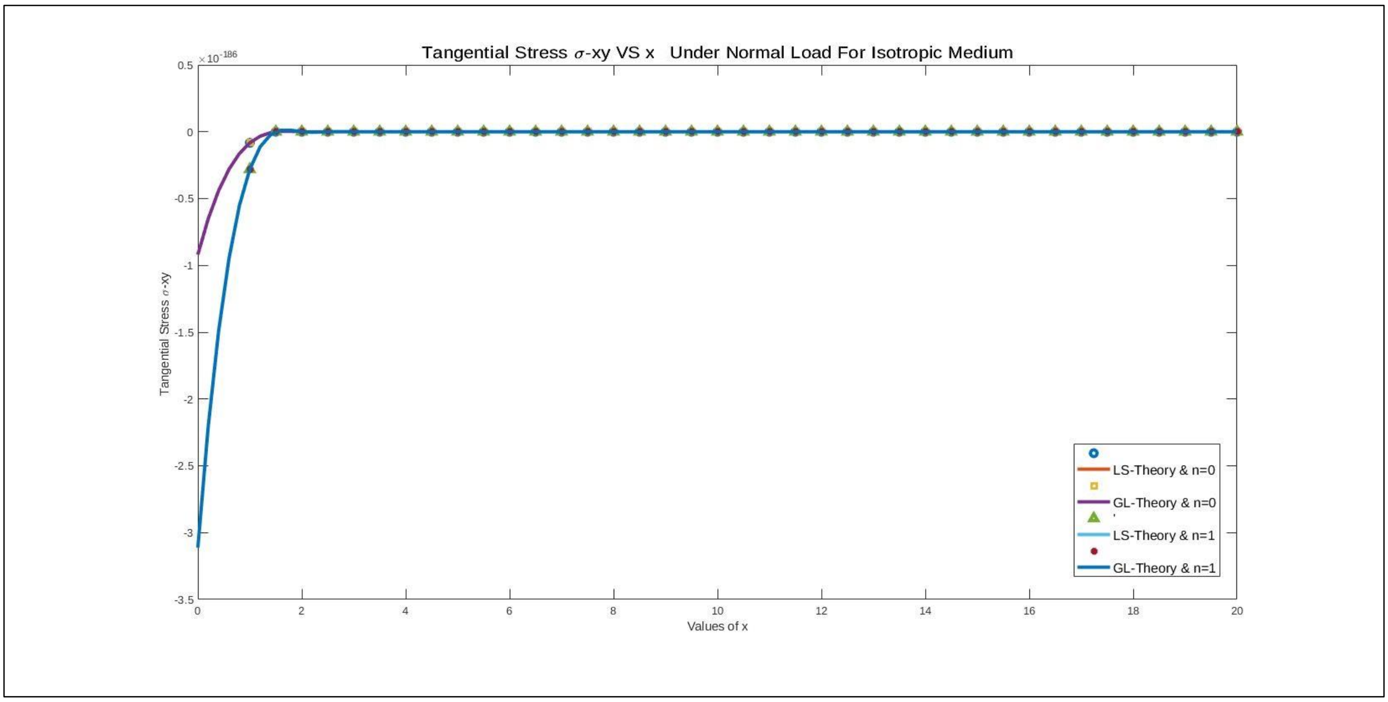
10
12
14
16
18
20


Normal Stress σ-xx VS x Under Normal Load For Isotropic Medium
×10^57
Normal Stress σ-yy
Values of x
LS-Theory & n=0
GL-Theory & n=0
LS-Theory & n=1
GL-Theory & n=1


Tangential Stress σ-xy VS x Under Normal Load For Isotropic Medium
×10^-186
Tangential Stress σ-xy
Values of x
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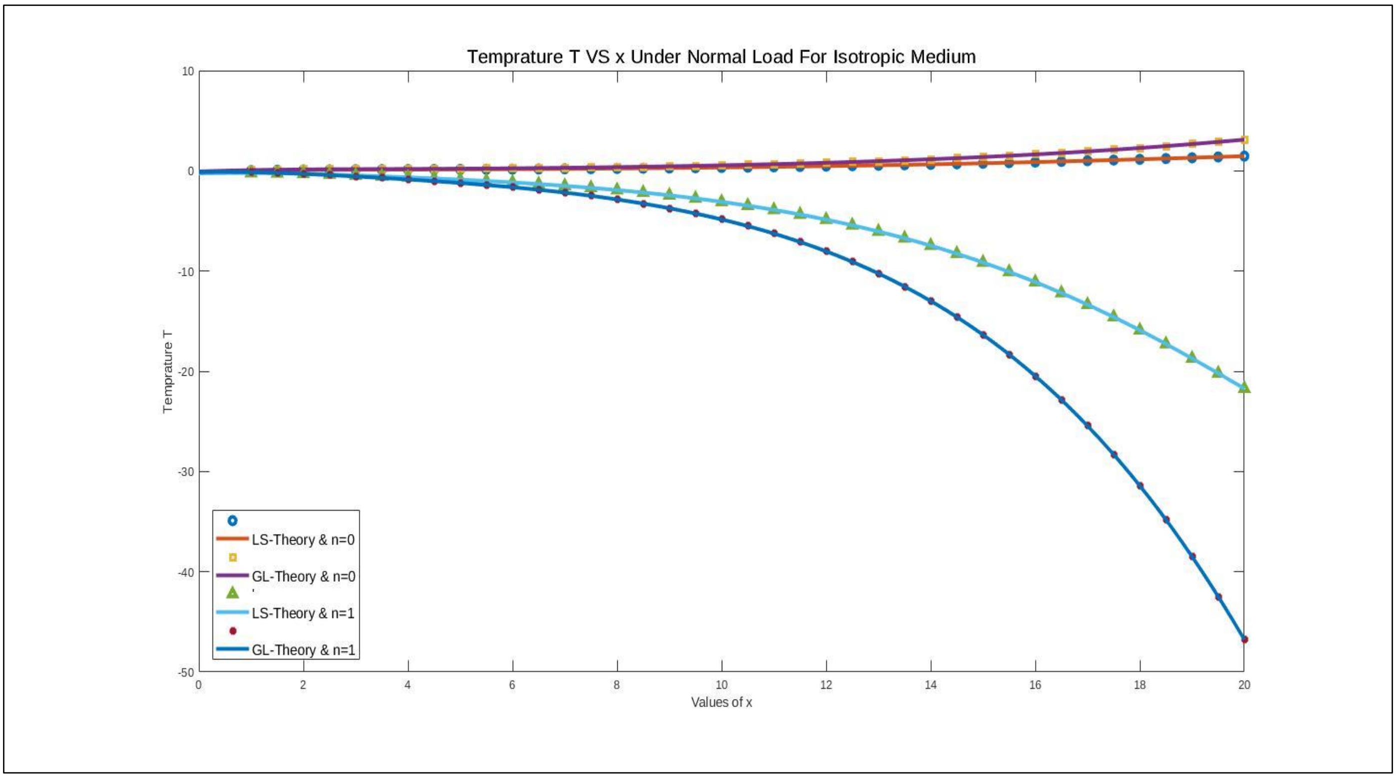
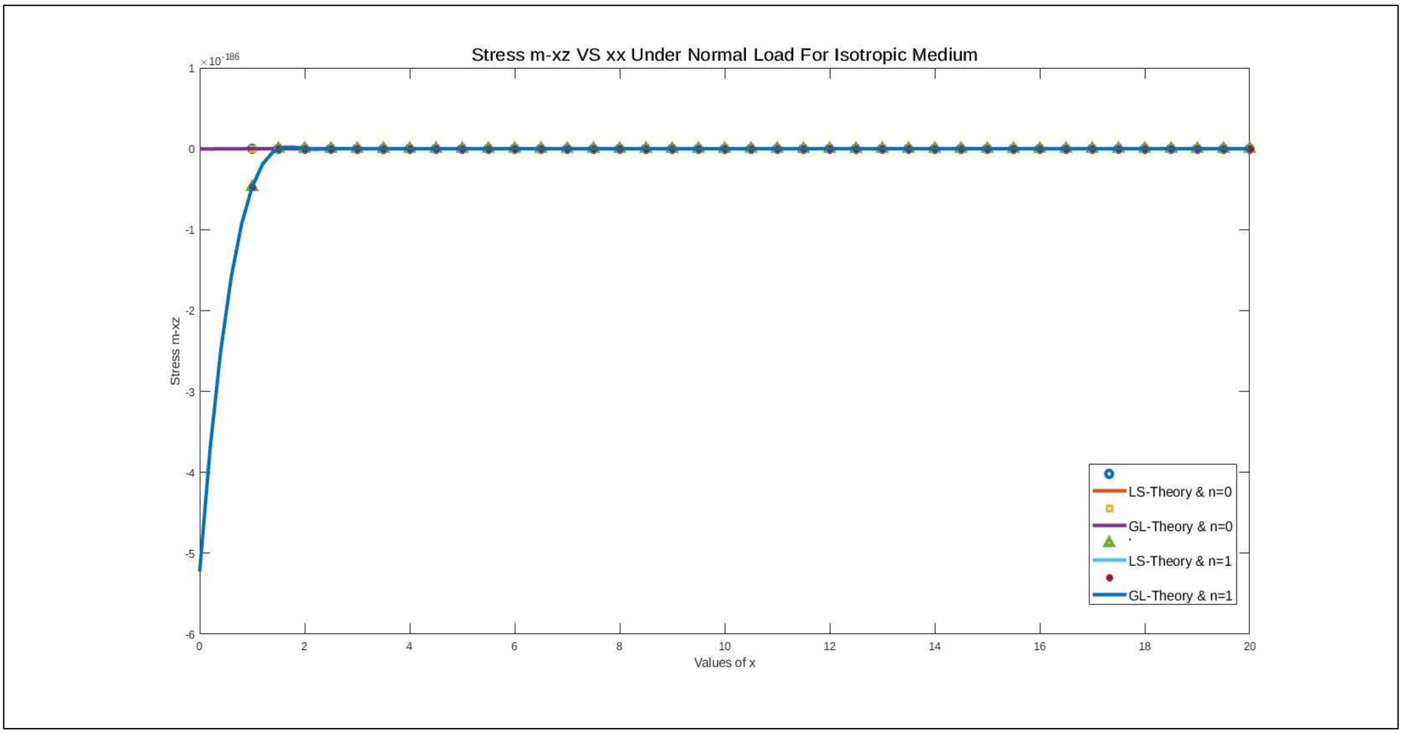
GL-Theory & n=0
LS-Theory & n=1
GL-Theory & n=1


Temprature T VS x Under Normal Load For Isotropic Medium
Temprature T
Values of x
10
0
-10
-20
-30
-40
-50
0
2
4
6
8
10
12
14
16
18
20
LS-Theory & n=0
GL-Theory & n=0
LS-Theory & n=1
GL-Theory & n=1


Stress m-xz VS xx Under Normal Load For Isotropic Medium
$\times 10^{-186}$
Stress m-xz
Values of x
1
0
-1
-2
-3
-4
-5
-6
0
2
4
6
8
10
12
14
16
18
20
LS-Theory & n=0
GL-Theory & n=0
LS-Theory & n=1
GL-Theory & n=1